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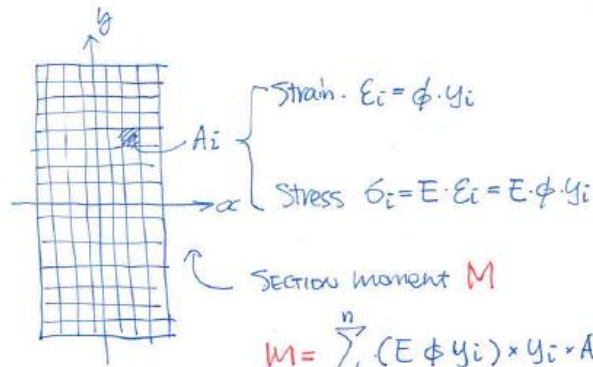
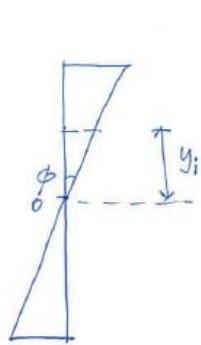
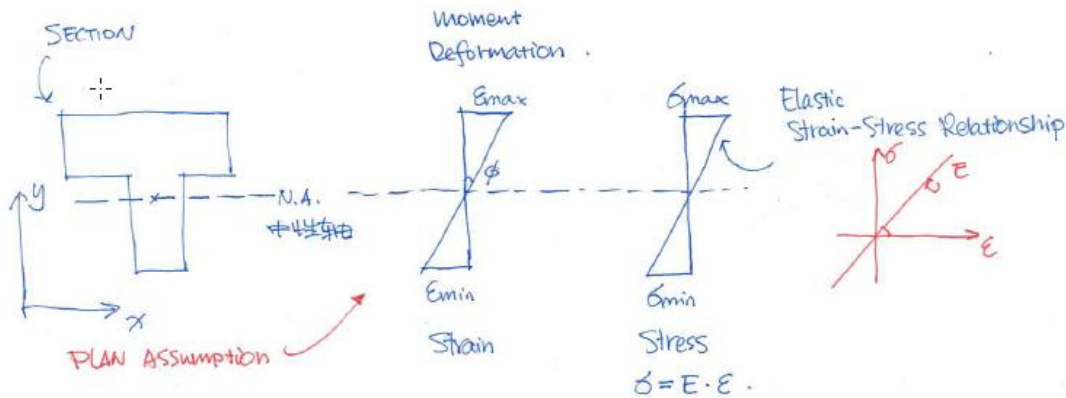
DinoStru 结构笔记课程

DINOSTRU 公开课，第五课

惯性矩的概念及计算方法

Concept and Calculation of Section Moment Inertia

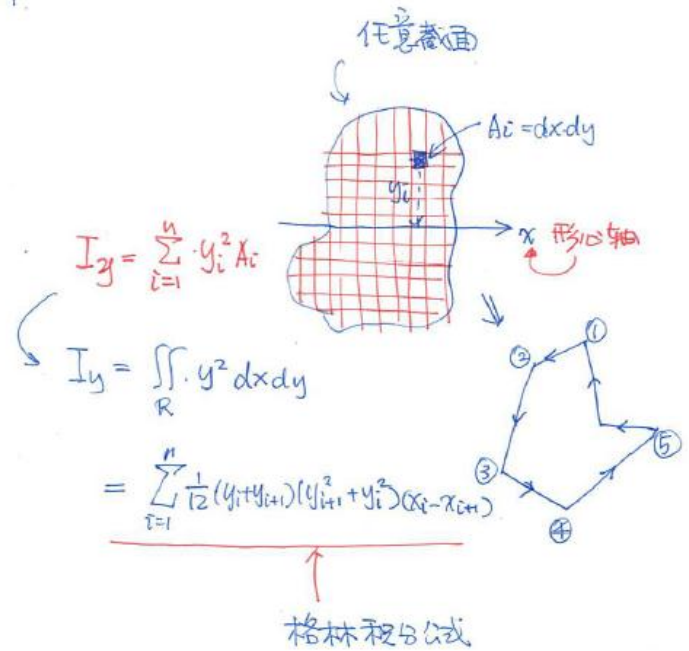
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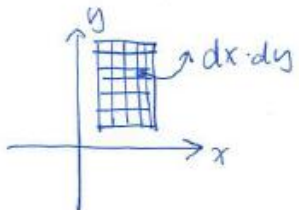
$$M = \sum_{i=1}^n (E \phi y_i) \times y_i \times A_i$$

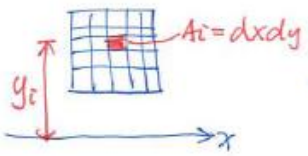
$$= \sum_{i=1}^n E \phi y_i^2 A_i = \phi \cdot E \cdot \sum_{i=1}^n y_i^2 A_i = \phi \cdot E \cdot I_y$$

$$I_y = \frac{M}{\phi E} \Rightarrow \phi = \frac{M}{E I_y}$$

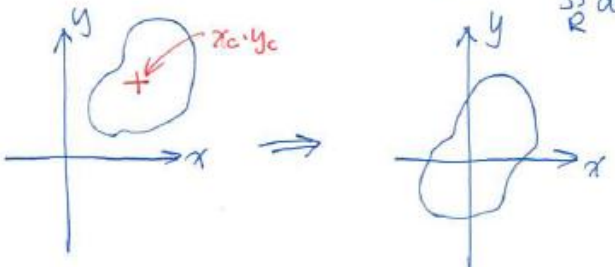


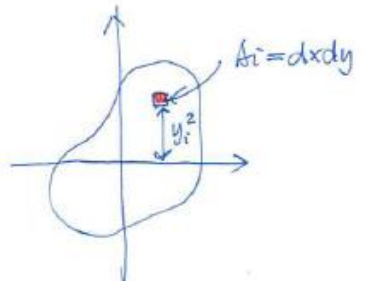
格林积分公式

$\iint_R dx dy$

 $\Rightarrow \iint_R dx dy = \text{Area} \text{ 总面积}$

$\iint_R x \cdot dx dy$
 $\iint_R y \cdot dx dy$

 $\Rightarrow c_x = \frac{\iint_R y dx dy}{\iint_R dx dy}$ 求形心

$c_y = \frac{\iint_R x dx dy}{\iint_R dx dy}$

$\begin{cases} x_j = x_i - x_c \\ y_j = y_i - y_c \end{cases}$

 \Rightarrow 移形心至几何轴心


 $\iint_R y^2 dx dy = I_y = \sum_{i=1}^n \frac{1}{12} \cdot (y_i + y_{i+1}) \cdot (y_i^2 + y_{i+1}^2) \cdot (x_i - x_{i+1})$

移形心后新坐标

Num	X1	Y1	AA	BB	CC
1	0	0	0	0	0
2	0	250	-125000	0	-15625000
3	500	250	0	43750000	0
4	500	600	-222000	0	-66600000
5	870	600	0	-227070000	0
6	870	0	0	0	0
1	0	0			

Sum	-347000	-183320000	-82225000
xc	528.2997		
yc	236.9597		

Num	X2	Y2	DD	EE
1	-528.3	-236.96	-1.229E+10	0.000E+00
2	-528.3	13.04035	0.000E+00	-3.696E+05
3	-28.2997	13.04035	-2.644E+06	0.000E+00
4	-28.2997	363.0403	0.000E+00	-5.901E+09
5	341.7003	363.0403	-7.979E+09	0.000E+00
6	341.7003	-236.96	0.000E+00	-3.859E+09
1	-528.3	-236.96		

Sum	-2.027E+10	-9.760E+09
lxx	2.027E+10	
lyy	9.760E+09	

$$\iint_R dx dy = \sum_{i=1}^n \frac{1}{2} (y_i + y_{i+1})(x_i - x_{i+1})$$

AA

$$\iint_R x dx dy = \sum_{i=1}^n \frac{1}{6} (x_i^2 + x_i x_{i+1} + x_{i+1}^2)(y_{i+1} - y_i)$$

BB

$$\iint_R y dx dy = \sum_{i=1}^n \frac{1}{6} (y_i^2 + y_i y_{i+1} + y_{i+1}^2)(x_i - x_{i+1})$$

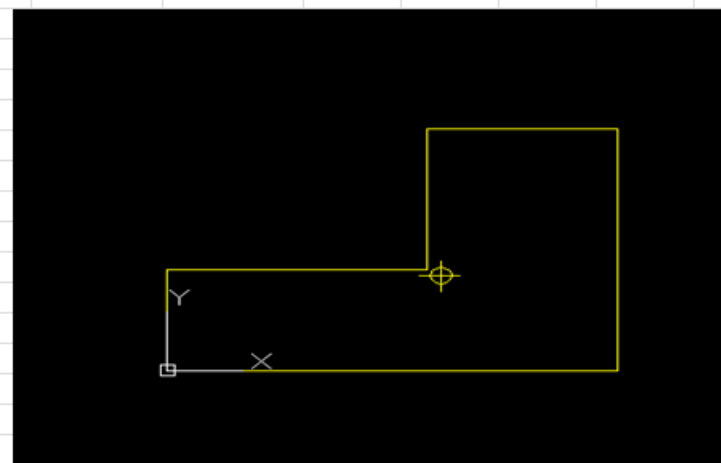
CC

$$\iint_R x^2 dx dy = \sum_{i=1}^n \frac{1}{12} (x_i + x_{i+1})(x_i^2 + x_{i+1}^2)(y_{i+1} - y_i)$$

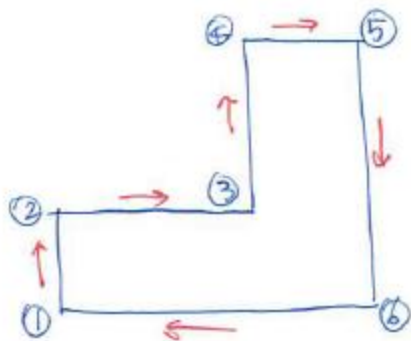
DD

$$\iint_R y^2 dx dy = \sum_{i=1}^n \frac{1}{12} (y_i + y_{i+1})(y_i^2 + y_{i+1}^2)(x_i - x_{i+1})$$

EE



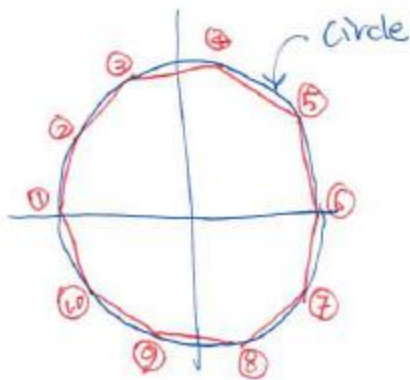
Sample ①



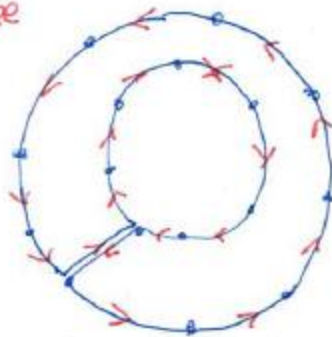
PROCEDURE

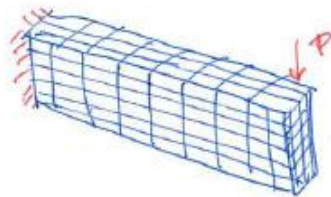
- ① 求形心 x_c, y_c
- ② 修改坐标 $x, y \rightarrow x_i, y_i$ 以形心为轴心
- ③ 求 $\iint_R y^2 dx dy$ 与 $\iint_R x^2 dx dy$ 得 I_y, I_x

注意



tube shape





Simplify

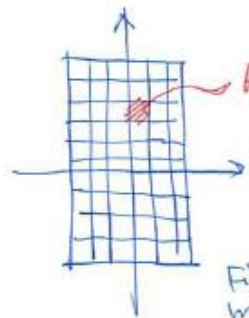


$$K = \frac{M}{\phi} = M \cdot E \cdot I_y$$

Material properties

Section properties

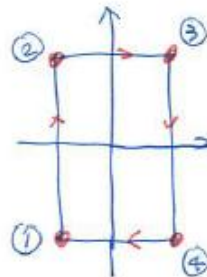
- ① 平截面假定
- ② 纤维模型
- ③ 数值积分 (格林积分)



A_i, E_i, σ_i

$$I_y = \sum_{i=1}^n y_i^2 A_i$$

Fiber model



$$I_y = \int_{\Omega} y^2 dx dy$$