

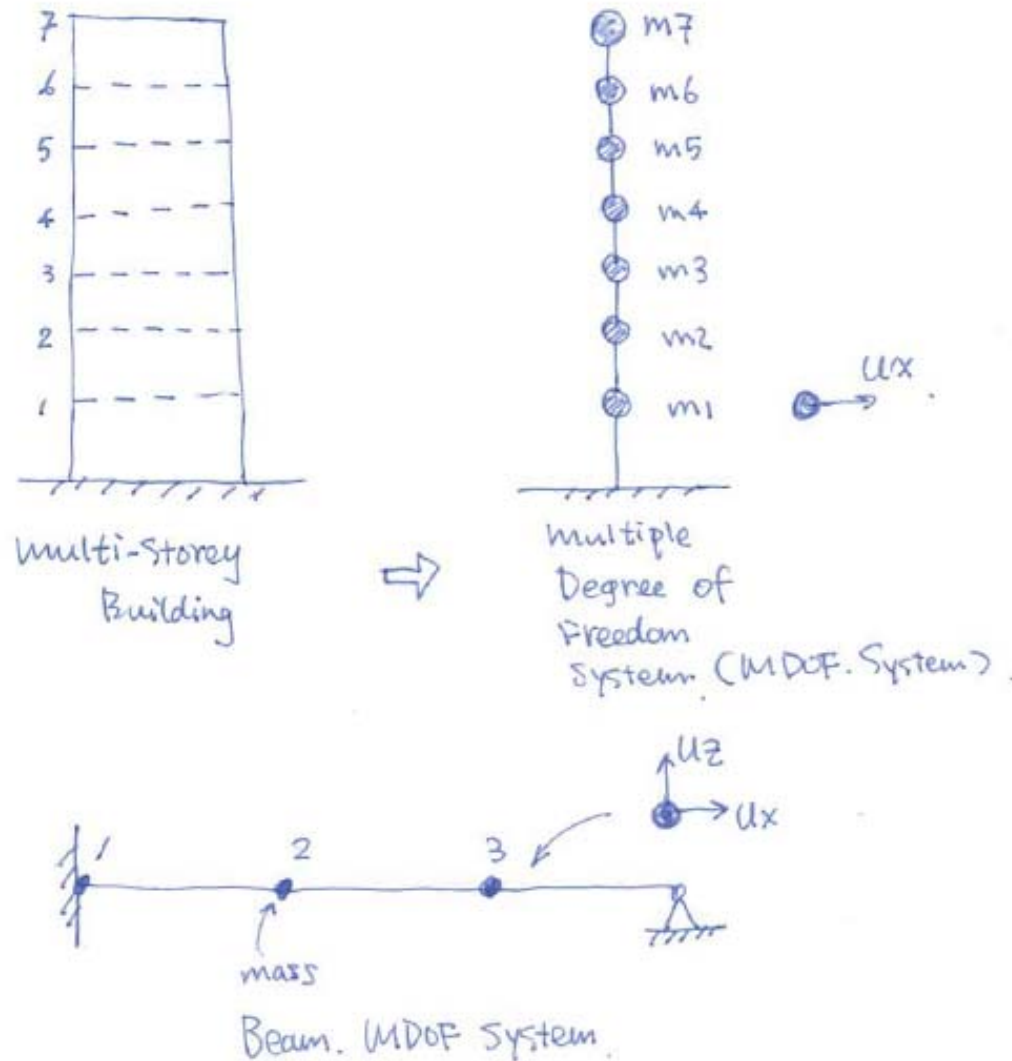
CIVL7008 Seismic Analysis for Building Structures

Lec-04 Modal Analysis of MDOF System

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Dr. Dino Chen

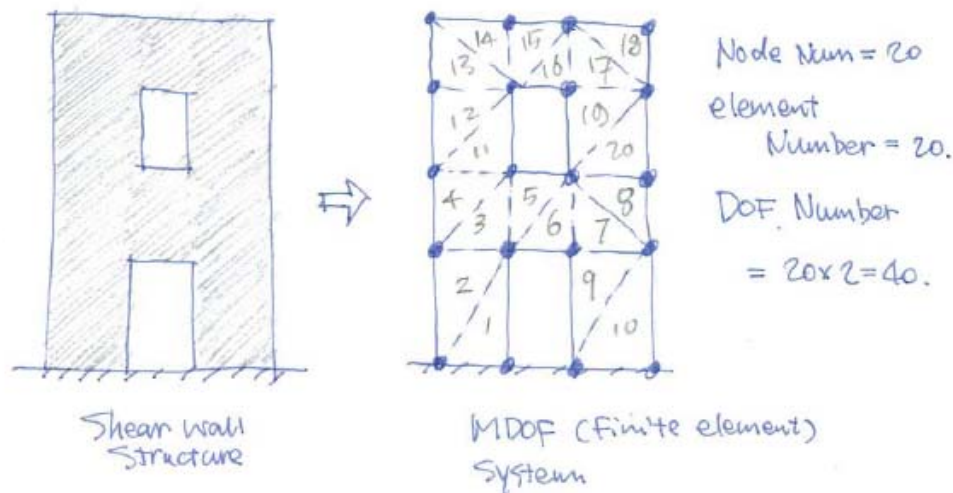
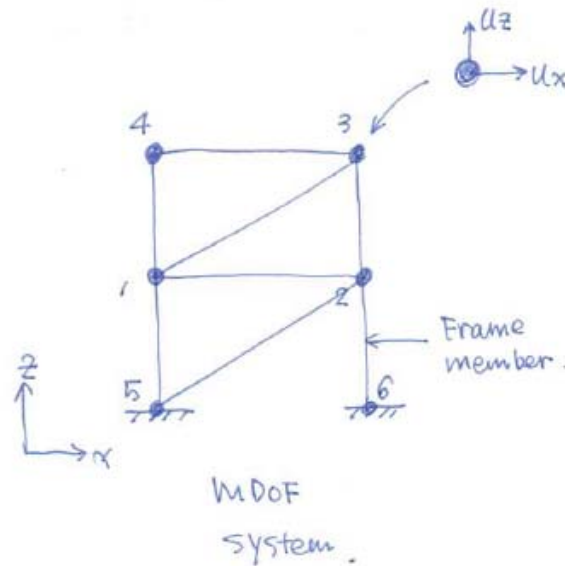
What is MDOF System ?



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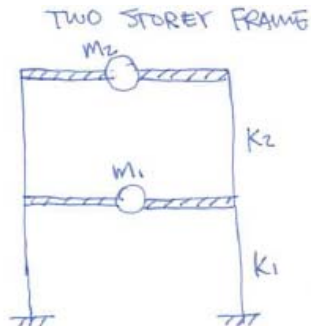
What is MDOF System ?



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Modal Analysis of 2 DOF System



① undamped system + free vibration.

$$[M] \cdot \{\ddot{u}\} + [K] \cdot \{u\} = \{0\}$$

② Assume vibration disp.

$$\{u\} = \{\phi\} \cdot \sin \omega t.$$

$$\textcircled{3} \quad \{\ddot{u}\} = -\omega^2 \cdot \{\phi\} \cdot \sin \omega t.$$

$$\textcircled{4} \quad [M] \cdot (-\omega^2) \cdot \{\phi\} \cdot \sin \omega t + [K] \cdot \{\phi\} \cdot \sin \omega t = \{0\}$$

$$\rightarrow ([K] - \omega^2 \cdot [M]) \cdot \{\phi\} = \{0\} \rightarrow \text{Homogeneous system of linear equations.}$$

$$\rightarrow \begin{vmatrix} [K] - \omega^2 \cdot [M] \end{vmatrix} = 0 \leftarrow \begin{array}{l} \text{Necessary Condition.} \\ \text{For nonzero solution.} \end{array}$$

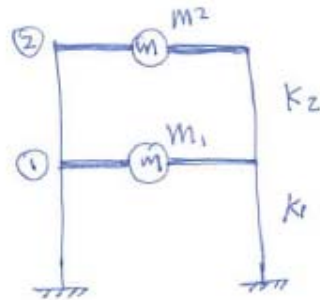
$$\rightarrow \begin{vmatrix} \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} - \omega^2 \cdot \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \end{vmatrix} = 0$$

$$\rightarrow \begin{vmatrix} k_{11} - \omega^2 m_1 & k_{12} \\ k_{21} & k_{22} - \omega^2 m_2 \end{vmatrix} = 0 \rightarrow (k_{11} - \omega^2 m_1) \cdot (k_{22} - \omega^2 m_2) - (k_{21} \cdot k_{12}) = 0 \rightarrow \begin{cases} \omega_1 \\ \omega_2 \end{cases}$$

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Example.

Storey mass. $m_1 = 60t$ $m_2 = 50t$

Storey lateral stiffness

$$k_1 = 5 \times 10^4 \text{ kN/m}$$

$$k_2 = 3 \times 10^4 \text{ kN/m}$$

Compute First & Second Mode period.
& mode shape.

$$[M] = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \quad [K] = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}$$

$$([K] - \omega^2 [M]) \{\phi\} = 0$$

$$|[K] - \omega^2 [M]| = 0 \rightarrow \begin{vmatrix} k_1 + k_2 - m_1 \omega^2 & -k_2 \\ -k_2 & k_2 - m_2 \omega^2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 8 \times 10^4 - 60 \omega^2 & -3 \times 10^4 \\ -3 \times 10^4 & 3 \times 10^4 - 50 \omega^2 \end{vmatrix} = 3 \omega^4 - 5.8 \times 10^3 \omega^2 + 1.5 \times 10^6 = 0$$

$$\Rightarrow 3A^2 - 5.8 \times 10^3 A + 1.5 \times 10^6 = 0$$

$$\Rightarrow \begin{cases} A_1 = \omega_1^2 = 307.6 & \omega_1 = 17.54 \text{ rad/s} \\ A_2 = \omega_2^2 = 1625.8 & \omega_2 = 40.32 \text{ rad/s} \end{cases}$$

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$$[K] - \omega^2 [M] \cdot \{\phi\} = \{0\}$$

$$\left(\begin{bmatrix} 8 \times 10^4 & -3 \times 10^4 \\ -3 \times 10^4 & 3 \times 10^4 \end{bmatrix} - \omega^2 \begin{bmatrix} 60 & 0 \\ 0 & 50 \end{bmatrix} \right) \cdot \{\phi\} = \{0\}$$

$$\omega_1^2 = 307.6$$

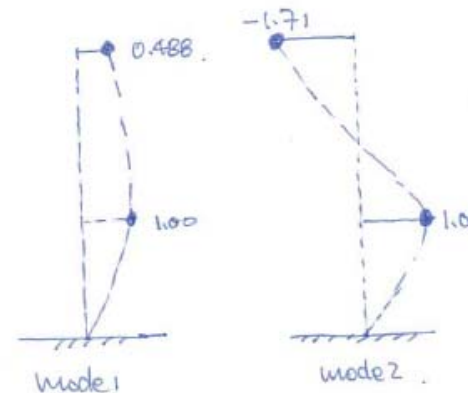
$$\Rightarrow \begin{bmatrix} 8 \times 10^4 - 307.6 \times 60 & -3 \times 10^4 \\ -3 \times 10^4 & 3 \times 10^4 - 50 \times 307.6 \end{bmatrix} \{\phi\} = \{0\}$$

$$\Rightarrow \begin{bmatrix} 6.15 & -3 \\ -3 & 1.46 \end{bmatrix} \cdot \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \{0\}$$

$$\Rightarrow \begin{cases} 6.15x_1 - 3x_2 = 0 \\ -3x_1 + 1.46x_2 = 0 \end{cases} \Rightarrow x_1 = \frac{3x_2}{6.15} = 0.488x_2$$

$$\{\phi_1\} = \begin{Bmatrix} 0.488x_2 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0.488 \\ 1 \end{Bmatrix} \rightarrow \text{First Mode Shape}$$

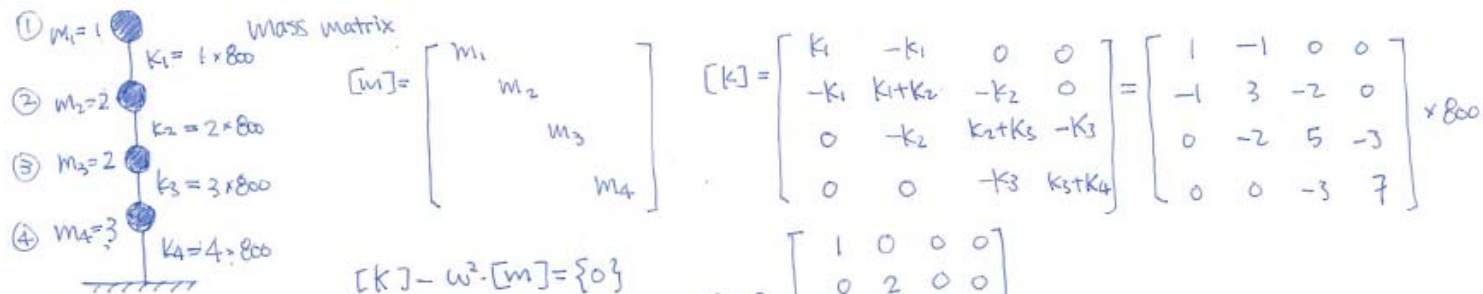
$$\text{SAME PROCESS. } \{\phi_2\} = \begin{Bmatrix} -1.71 \\ 1 \end{Bmatrix} \rightarrow \text{Second Mode Shape}$$



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Example. 4-DOF-System. Mode Shape Calculation.



$$[K] - \omega^2 \cdot [m] = \{0\}$$

$$\rightarrow |[K] - \omega^2 \cdot [m]| = 0$$

$$[m] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

$$|[K] - \omega^2 \cdot [m]| = \begin{vmatrix} 800 - \omega^2 & -800 & 0 & 0 \\ -800 & 2400 - 2\omega^2 & -1600 & 0 \\ 0 & -1600 & 4000 - 2\omega^2 & -2400 \\ 0 & 0 & -2400 & 5600 - 3\omega^2 \end{vmatrix}$$

Determinant:

$$\begin{vmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{vmatrix} = a \begin{vmatrix} f & g & h \\ j & k & l \\ n & o & p \end{vmatrix} - b \begin{vmatrix} e & g & h \\ i & k & l \\ m & o & p \end{vmatrix} + c \begin{vmatrix} e & f & h \\ i & j & l \\ m & n & p \end{vmatrix} - d \begin{vmatrix} e & f & g \\ i & j & k \\ m & n & o \end{vmatrix}$$

$$= (800 - \omega^2) \cdot \begin{vmatrix} 2400 - 2\omega^2 & -1600 & 0 \\ -1600 & 4000 - 2\omega^2 & -2400 \\ 0 & -2400 & 5600 - 3\omega^2 \end{vmatrix} - (-800) \cdot \begin{vmatrix} -800 & -1600 & 0 \\ 0 & 4000 - 2\omega^2 & -2400 \\ 0 & -2400 & 5600 - 3\omega^2 \end{vmatrix}$$

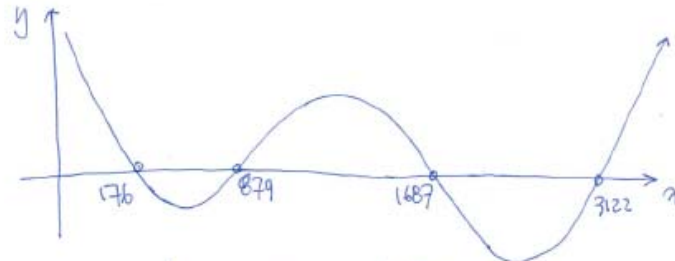
$$= (800 - \omega^2) \cdot [(2400 - 2\omega^2)(4000 - 2\omega^2)(5600 - 3\omega^2) - (-2400)(-2400)(2400 - 2\omega^2) - (-1600)(-1600)(5600 - 3\omega^2)]$$

$$- (-800) \cdot [(-800) \cdot (4000 - 2\omega^2)(5600 - 3\omega^2) - (-2400)(-2400)(-800)] = 0$$

$$y = f(\omega^2) = 0; \text{ USE EXCEL TO compute } \omega^2.$$

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 $x = \omega^2$ $y = f(x) = 0$ EXCEL Table Calculation.

$x_1 = 176$	$\omega_1 = 13.2$	$T_1 = 0.474$
$x_2 = 876$	$\omega_2 = 29.65$	$T_2 = 0.212$
$x_3 = 1687$	$\omega_3 = 41.07$	$T_3 = 0.153$
$x_4 = 3122$	$\omega_4 = 55.87$	$T_4 = 0.112$

Mode Shape Calculation. $\omega_1^2 = 176$. (mode shape 1)

$$D(\omega_1^2) = \begin{bmatrix} 800-176 & -800 & 0 & 0 \\ -800 & 2400-752 & -1600 & 0 \\ 0 & -1600 & 4000-352 & -2400 \\ 0 & 0 & -2400 & 5600-528 \end{bmatrix} = \begin{bmatrix} 624 & -800 & 0 & 0 \\ -800 & 2048 & -1600 & 0 \\ 0 & -1600 & 3648 & -2400 \\ 0 & 0 & -2400 & 5072 \end{bmatrix}$$

$$D_{bb} = \begin{bmatrix} 2048 & -1600 & 0 \\ -1600 & 3648 & -2400 \\ 0 & -2400 & 5072 \end{bmatrix} \quad D_{ba} = \begin{bmatrix} -800 \\ 0 \\ 0 \end{bmatrix}$$

$$\{\phi\}_b = -[D_{bb}]^{-1} \cdot D_{ba}$$

$$= -\frac{1}{12805312} \times \begin{Bmatrix} 12444 \times (-800) \\ 7925 \times (-800) \\ 3750 \times (-800) \end{Bmatrix} = \begin{Bmatrix} 0.777 \\ 0.495 \\ 0.234 \end{Bmatrix}$$

$$D_{bb}^{-1} = \frac{1}{12805312} \begin{bmatrix} 12444 & 7925 & 3750 \\ 7925 & 10144 & 4800 \\ 3750 & 4800 & 4796 \end{bmatrix}$$

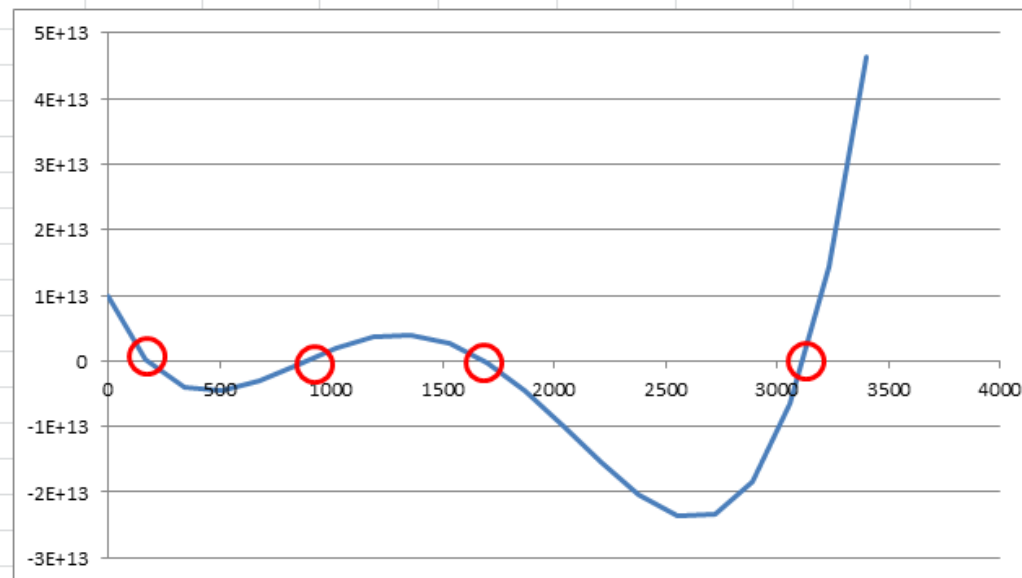
$$\Rightarrow \{\phi\}_1 = \begin{Bmatrix} 1.000 \\ 0.777 \\ 0.495 \\ 0.234 \end{Bmatrix}$$

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Use Excel table to Eigenvalue

	start	0
	inc	170
step	x	y
0	0	9.8304E+12
1	170	2.56339E+11
2	340	-3.96523E+12
3	510	-4.54876E+12
4	680	-2.96814E+12
5	850	-4.56725E+11
6	1020	1.99265E+12
7	1190	3.62772E+12
8	1360	3.93675E+12
9	1530	2.64852E+12
10	1700	-2.676E+11
11	1870	-4.60174E+12
12	2040	-9.90348E+12
13	2210	-1.54819E+13
14	2380	-2.04053E+13
15	2550	-2.35019E+13
16	2720	-2.3359E+13
17	2890	-1.83234E+13
18	3060	-6.50158E+12
19	3230	1.42408E+13
20	3400	4.62784E+13



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mode shape 2.

$$\omega_2^2 = 876$$

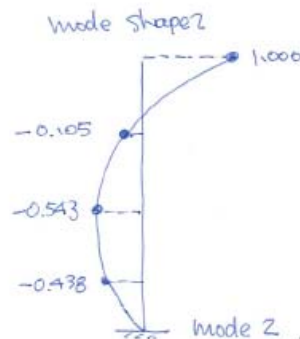
$$D(\omega_2) = \begin{bmatrix} -76 & -800 & 0 & 0 \\ -800 & 648 & -1600 & 0 \\ 0 & -1600 & 2248 & -2400 \\ 0 & 0 & -2400 & 2972 \end{bmatrix} = \begin{bmatrix} D_{aa} & D_{ab} \\ D_{ba} & D_{bb} \end{bmatrix}$$

$$D_{bb} = \begin{bmatrix} 648 & -1600 & 0 \\ -1600 & 2248 & -2400 \\ 0 & -2400 & 2972 \end{bmatrix} \quad D_{ab} = \begin{bmatrix} -800 \\ 0 \\ 0 \end{bmatrix}$$

$$D_{bb}^{-1} = \frac{1}{219108616} \begin{bmatrix} -28783 & -148600 & -120000 \\ -148600 & -60183 & -48600 \\ -120000 & -48600 & 34478 \end{bmatrix}$$

$$(\phi_2)_n = -D_{bb}^{-1} \cdot D_{ab} = \frac{1}{219108616} \times (-800) \times \begin{Bmatrix} -28783 \\ -148600 \\ -120000 \end{Bmatrix} = \begin{Bmatrix} -0.105 \\ -0.543 \\ -0.438 \end{Bmatrix}$$

$$\phi_2 = \begin{Bmatrix} 1.000 \\ -0.105 \\ -0.543 \\ -0.438 \end{Bmatrix}$$



mode shape matrix

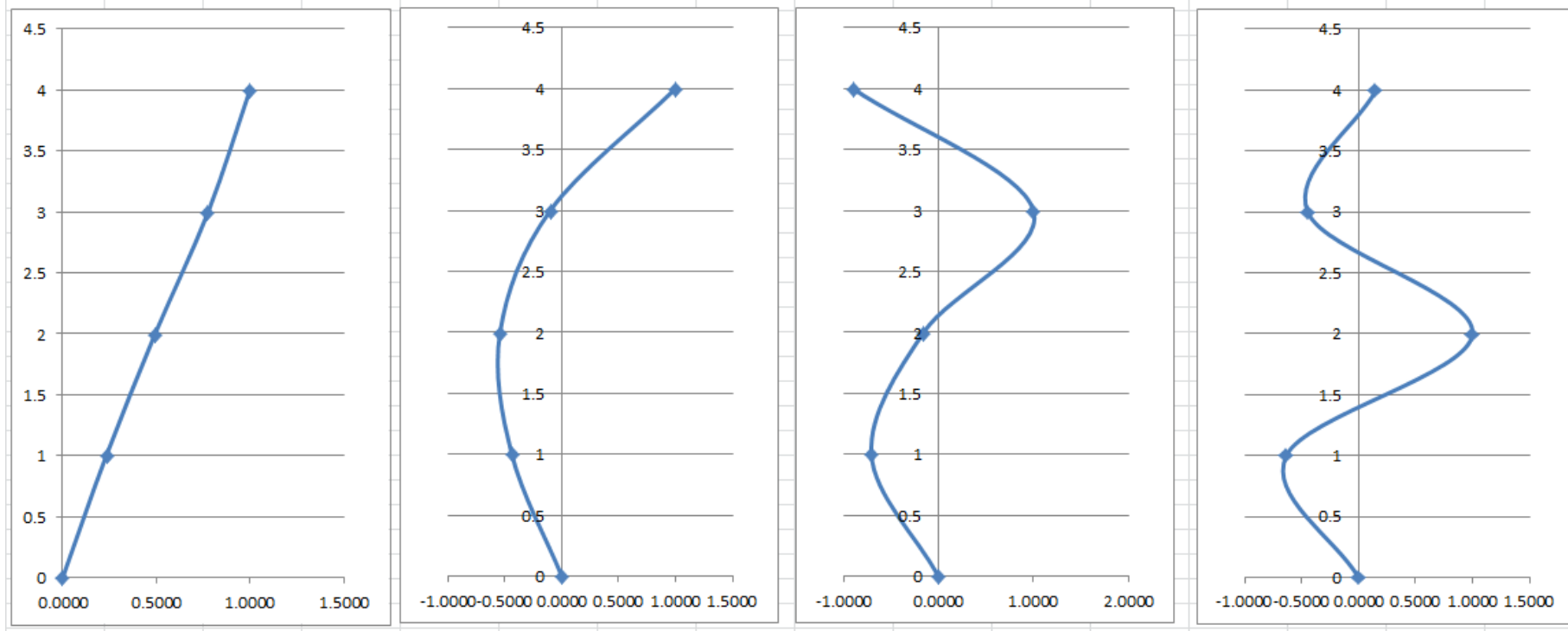
$$\phi = \begin{bmatrix} 1.000 & 1.000 & -0.90145 & 0.15436 \\ 0.79910 & -0.09963 & 1.000 & -0.44817 \\ 0.44655 & -0.53989 & -0.15859 & 1.0000 \\ 0.24506 & -0.43761 & -0.70797 & -0.63688 \end{bmatrix}$$

\uparrow mode 1 \uparrow mode 2 \uparrow mode 3 \uparrow mode 4

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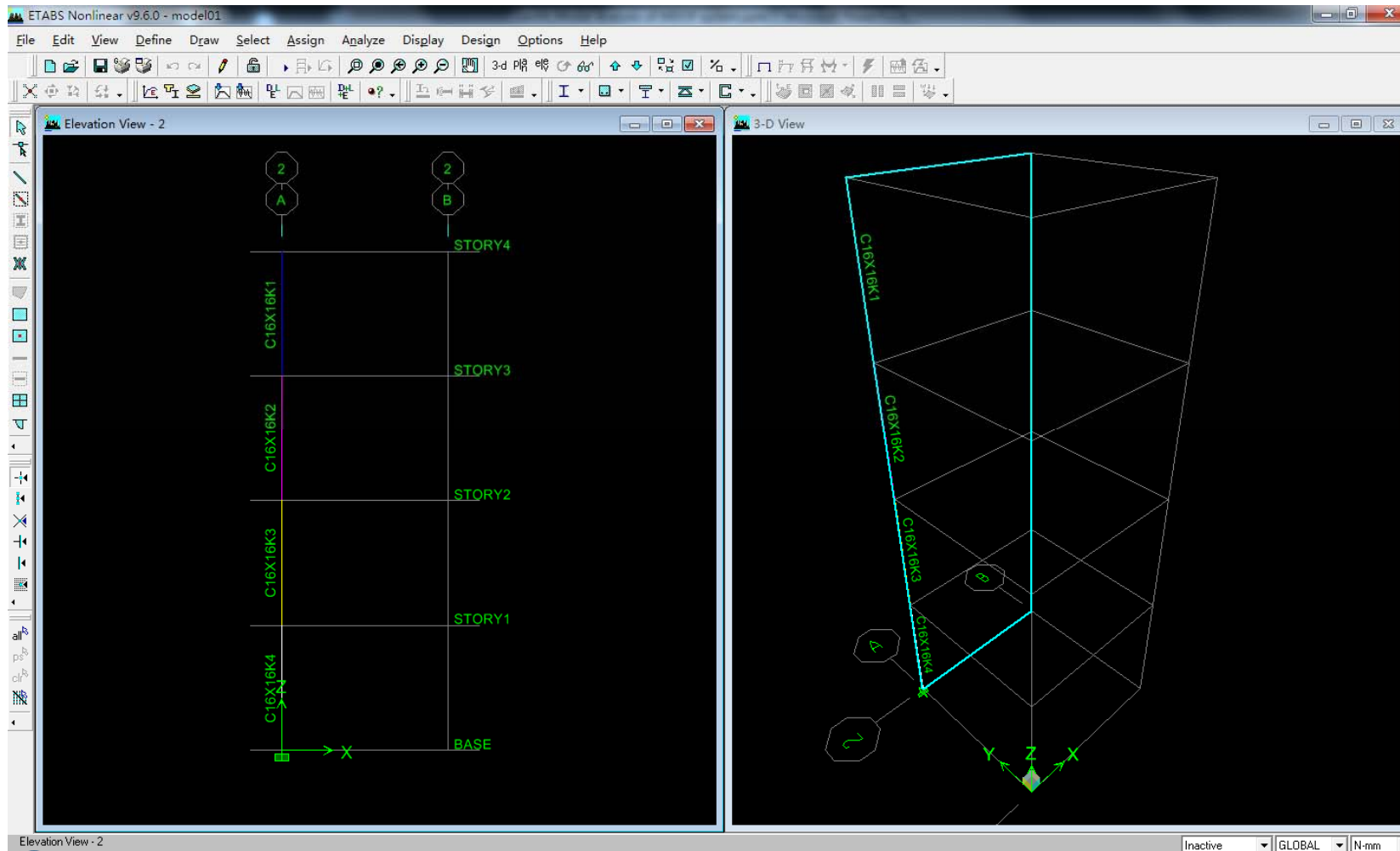
Use Excel to Plot Mode Shape



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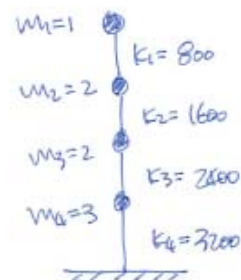
Etabs or SAP2000 model compute mode shape



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Etabs or SAP2000 model compute mode shape



Shear Stiffness.

$$k = \frac{GA_s}{L}$$

$$L = 3000 \text{ (mm)}$$

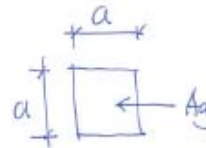
$$G = 10416.67 \text{ MPa}$$

$$A_s = 0.8333 A_g$$

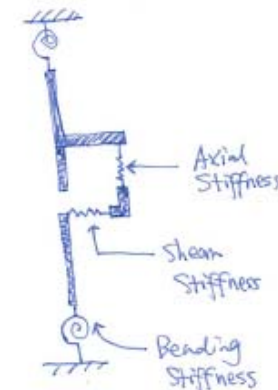
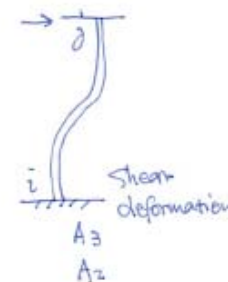
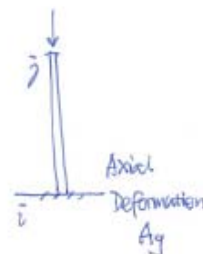
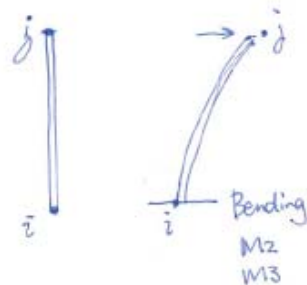
$$A_g = a^2$$

$$k_i = 800 \text{ (N/mm)}$$

$$a = \sqrt{A_g} = \sqrt{\frac{kL}{G} \times \frac{1}{0.8333}} = \sqrt{\frac{800 \times 3000}{10416.67 \times 0.8333}} = 16.628 \text{ (mm)}$$



$$\left\{ \begin{array}{ll} \text{Axial Area } A_g & = \times 10^8 \\ \text{Shear Area } A_3 & = \times 1 \\ \text{Shear Area } A_2 & = \times 1 \\ \text{Torsional Constant } T & \\ \text{Moment of Inertia About Axis 2} & = \times 10^8 \\ \text{Moment of Inertia About Axis 3} & = \times 10^8 \end{array} \right.$$



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Etabs or SAP2000 model compute mode shape

Define Frame Properties

Properties
Type in property to find:

B200x500
B200x500
C16x16K1
C16x16K2
C16x16K3
C16x16K4

Click to:
Import I/Wide Flange
Add I/Wide Flange
Modify/Show Property...
Delete Property

OK
Cancel

Analysis Property Modification Factors

Property Modifiers

Cross-section (axial) Area: 100000000
Shear Area in 2 direction: 3
Shear Area in 3 direction: 3
Torsional Constant: 1
Moment of Inertia about 2 axis: 100000000
Moment of Inertia about 3 axis: 100000000
Mass: 1
Weight: 1

Define Mass Source

Mass Definition
☒ From Self and Specified Mass
☐ From Loads
☐ From Self and Specified Mass and Loads

Define Mass Multiplier for Loads

Load	Multiplier
DEAD	1

☒ Include Lateral Mass Only
☒ Lump Lateral Mass at Story Levels

OK
Cancel

Rectangular Section

Section Name: C16x16K3

Properties: Section Properties...
Property Modifiers: Set Modifiers...
Material: C35

Dimensions
Depth (t3): 16.6278
Width (t2): 16.6278

Concrete
Reinforcement...

Display Color:

OK
Cancel

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Analysis Options

Building Active Degrees of Freedom

Full 3D XZ Plane YZ Plane No Z Rotation

☒ UX ☐ UY ☒ UZ ☐ RX ☒ RY ☐ RZ

☒ Dynamic Analysis Set Dynamic Parameters...

☐ Include P-Delta Set P-Delta Parameters...

☐ Save Access DB File File Name...

OK Cancel

Dynamic Analysis Parameters

Number of Modes 4

Type of Analysis

☒ Eigenvectors ☐ Ritz Vectors

EigenValue Parameters

Frequency Shift (Center) 0.

Cutoff Frequency (Radius) 0.

Relative Tolerance 1.000E-07

☐ Include Residual-Mass Modes

Starting Ritz Vectors

List of Loads Ritz Load Vectors

Add -> <- Remove

OK Cancel

Assign Masses

Masses in Global Directions

Direction X, Y 0.

Direction Z 0.

Mom. of Inertia in Global Directions

Rotation about X 0.

Rotation about Y 0.

Rotation about Z 0.

Options

☐ Add to Existing Masses

☒ Replace Existing Masses

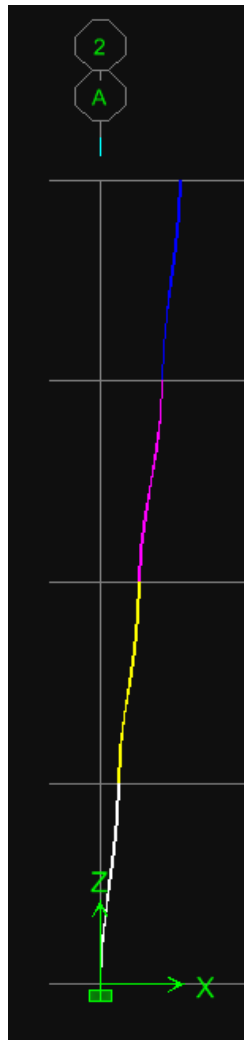
☐ Delete Existing Masses

OK Cancel

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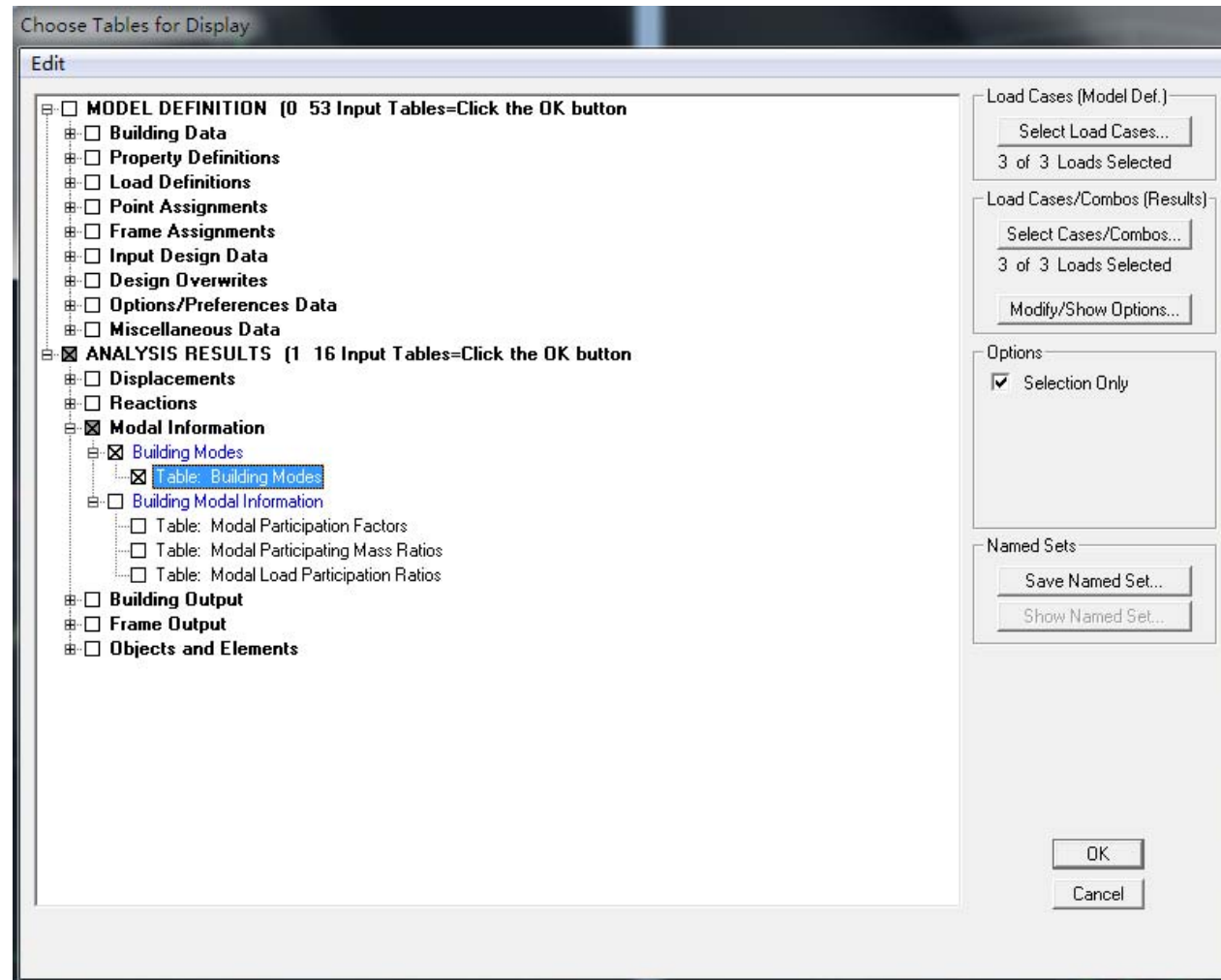
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Building Modes

Edit View

Building Modes

	Story	Diaphragm	Mode	UX	UY	UZ	RX	RY	RZ
▶	STORY4	D1	1	0.5917	0.0000	0.0000	0.00000	0.00000	0.00000
	STORY3	D1	1	0.4596	0.0000	0.0000	0.00000	0.00000	0.00000
	STORY2	D1	1	0.2921	0.0000	0.0000	0.00000	0.00000	0.00000
	STORY1	D1	1	0.1376	0.0000	0.0000	0.00000	0.00000	0.00000
	STORY4	D1	2	-0.6758	0.0000	0.0000	0.00000	0.00000	0.00000
	STORY3	D1	2	0.0685	0.0000	0.0000	0.00000	0.00000	0.00000
	STORY2	D1	2	0.3667	0.0000	0.0000	0.00000	0.00000	0.00000
	STORY1	D1	2	0.2972	0.0000	0.0000	0.00000	0.00000	0.00000
	STORY4	D1	3	-0.4321	0.0000	0.0000	0.00000	0.00000	0.00000
	STORY3	D1	3	0.4783	0.0000	0.0000	0.00000	0.00000	0.00000
	STORY2	D1	3	-0.0756	0.0000	0.0000	0.00000	0.00000	0.00000
	STORY1	D1	3	-0.3388	0.0000	0.0000	0.00000	0.00000	0.00000
	STORY4	D1	4	0.0808	0.0000	0.0000	0.00000	0.00000	0.00000
	STORY3	D1	4	-0.2351	0.0000	0.0000	0.00000	0.00000	0.00000
	STORY2	D1	4	0.5239	0.0000	0.0000	0.00000	0.00000	0.00000
	STORY1	D1	4	-0.3336	0.0000	0.0000	0.00000	0.00000	0.00000

◀ ▶

⏮ ⏪ ⏩ ⏭

OK

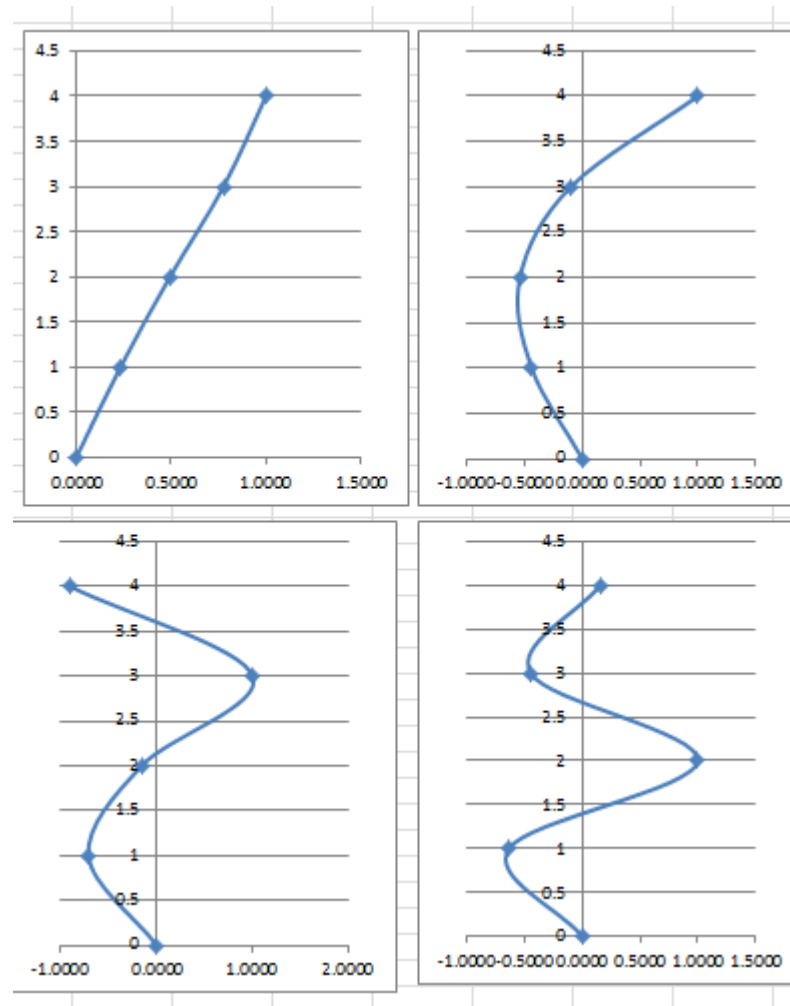
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Normalization

MODE	1	2	3	4
STORY4	0.5917	-0.6758	-0.4321	0.0808
STORY3	0.4596	0.0685	0.4783	-0.2351
STORY2	0.2921	0.3667	-0.0756	0.5239
STORY1	0.1376	0.2972	-0.3388	-0.3336
max	0.5917	0.3667	0.4783	0.5239
min	0.1376	-0.6758	-0.4321	-0.3336
absmax	0.5917	0.6758	0.4783	0.5239
sign	1	-1	1	1
maxv	0.5917	-0.6758	0.4783	0.5239

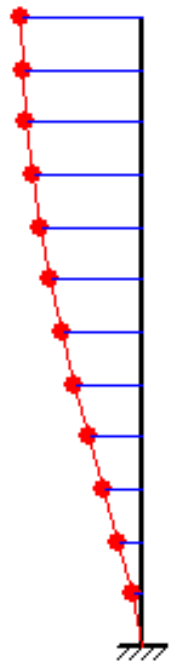
Storey	mode 1	mode 2	mode 3	mode 4
4	1.0000	1.0000	-0.9034	0.1542
3	0.7767	-0.1014	1.0000	-0.4487
2	0.4937	-0.5426	-0.1581	1.0000
1	0.2326	-0.4398	-0.7083	-0.6368
0	0.0000	0.0000	0.0000	0.0000



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How about more than 4 storey Building (Simplified model) ?



Transform the Matrix D to Matrix B

$$[M]\{\ddot{x}(t)\} + [K]\{\dot{x}(t)\} = \{0\}$$

Assume $[M]^{-1}[K] = \{P\}$

$$\left. \begin{aligned} ([P] - \omega^2 [I])\{X\} &= \{0\} \\ [P]\{X\} &= \omega^2 \{X\} \end{aligned} \right\}$$

Take

$$\{Z\} = [\sqrt{M}]\{X\}$$

$$\begin{vmatrix} P_{11} - \omega^2 & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} - \omega^2 & \cdots & P_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ P_{n1} & P_{n2} & \cdots & P_{nn} - \omega^2 \end{vmatrix} = 0$$

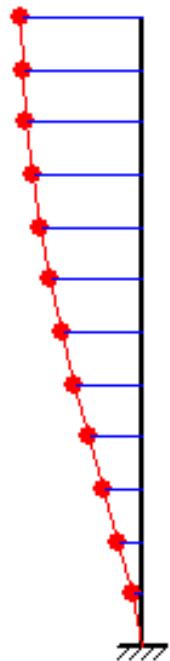
$$[B]\{Z\} = \omega^2 \{Z\}$$

$$[B] = [\sqrt{M}]^{-1}[K][\sqrt{M}]^{-1}$$

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How about more than 4 storey Building (Simplified model) ?



$$[B] = \begin{bmatrix} \frac{K_1 + K_2}{m_1} & \frac{-K_2}{\sqrt{m_1}\sqrt{m_2}} & 0 & \dots & 0 \\ \frac{-K_2}{\sqrt{m_1}\sqrt{m_2}} & \frac{K_2 + K_3}{m_2} & \frac{-K_3}{\sqrt{m_2}\sqrt{m_3}} & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \frac{-K_{n-1}}{\sqrt{m_{n-2}}\sqrt{m_{n-1}}} & \frac{K_{n-1} + K_n}{m_{n-1}} & \frac{-K_n}{\sqrt{m_{n-1}}\sqrt{m_n}} \\ 0 & 0 & \dots & \frac{-K_n}{\sqrt{m_{n-1}}\sqrt{m_n}} & \frac{K_n}{m_n} \end{bmatrix}$$

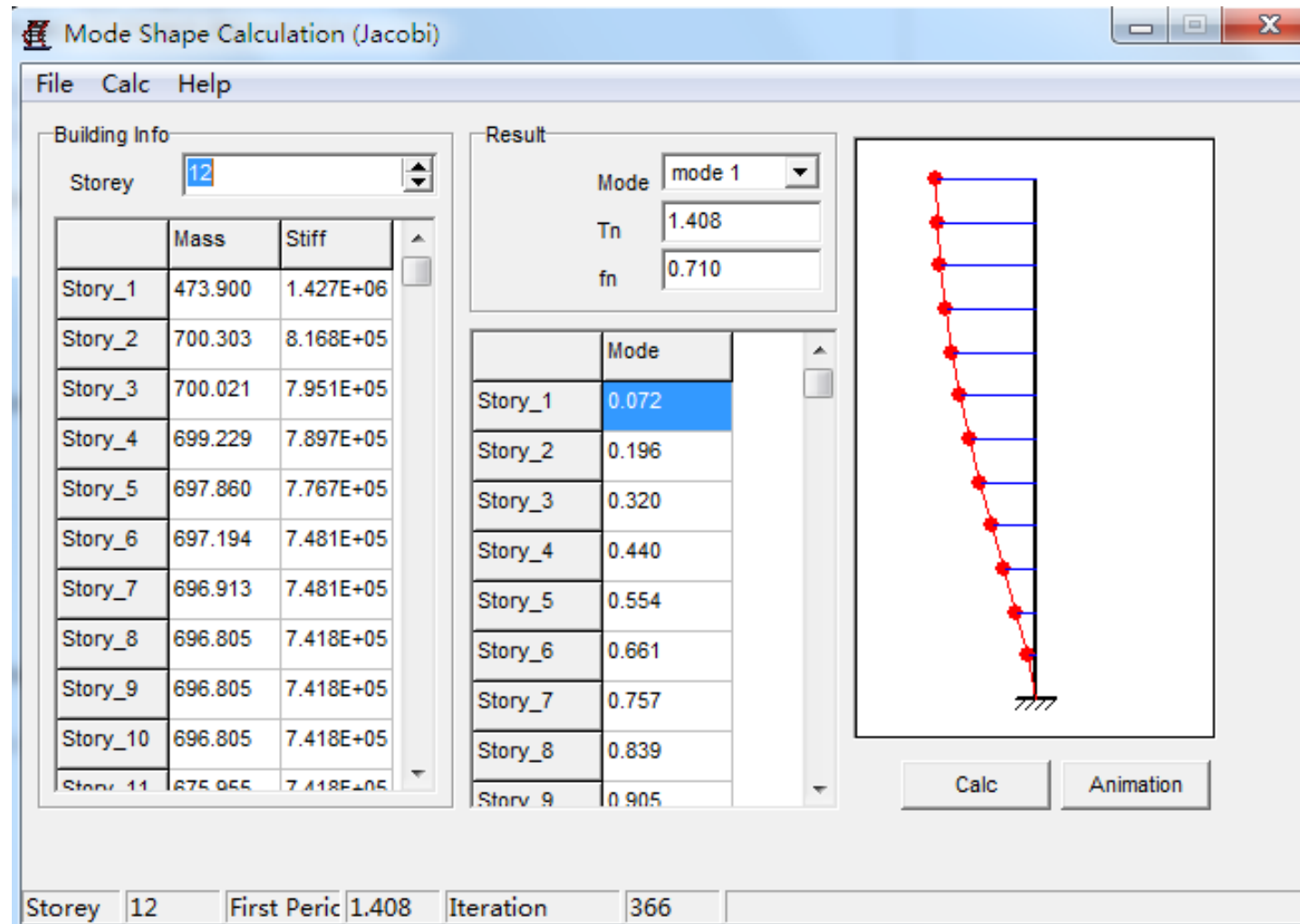
Relationship between $\{x\}$ and $\{z\}$, which use to compute mode shape

$$\{X\} = [\sqrt{M}]^{-1} \{Z\}$$

Lec-04 Modal Analysis of MDOF system

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Mode Shape Calculation Program



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Mode Shape Calculation Program

```
//form matrix K and Matrix M
for i:=1 to n do
begin
  k[i]:=strtofloat(stringgrid1.Cells[2,i]);
  m[i]:=strtofloat(stringgrid1.Cells[1,i]);
end;
//Form Matrix B
setlength(b,n+1,n+1);
setlength(v,n+1,n+1);
setlength(m1,n+1,n+1);
setlength(rr,n+1,n+1);
setlength(rr2,n+1,n+1);
for i:=1 to n do
for j:=1 to n do
begin
  b[i,j]:=0; //Clear
end;
for i:=2 to n-1 do
begin
  b[i,i]:=(k[i]+k[i+1])/m[i];
  b[i,i-1]:=-k[i]/(sqrt(m[i-1])*sqrt(m[i]));
  b[i,i+1]:=-k[i+1]/(sqrt(m[i])*sqrt(m[i+1]));
end;
b[1,1]:=(k[1]+k[2])/m[1];
b[1,2]:=-k[2]/(sqrt(m[1])*sqrt(m[2]));
b[n,n]:=k[n]/m[n];
b[n,n-1]:=-k[n]/(sqrt(m[n-1])*sqrt(m[n]));
//Form Matrix B
//
combobox1.Clear;
for i:=1 to n do
begin
  combobox1.Items.Add(format('mode %d',[i]));
end;
```

```
jacobi(b,n,d,v,nrot);
```

```
statusbar1.Panels[5].Text:=inttostr(nrot);
for i:=1 to n do
zx[i]:=2*pi/sqrt(d[i]);
for i:=1 to n do
num[i]:=i;
for j:=1 to n do
for i:=1 to n-j do
begin
  if zx[i]<zx[i+1] then
  begin
    temp:=zx[i];
    temp2:=num[i];
    zx[i]:=zx[i+1];
    num[i]:=num[i+1];
    zx[i+1]:=temp;
    num[i+1]:=temp2;
  end;
end; //order
```

$$[B] = \begin{bmatrix} \frac{K_1+K_2}{m_1} & \frac{-K_2}{\sqrt{m_1}\sqrt{m_2}} & 0 & \dots & 0 \\ \frac{-K_2}{\sqrt{m_1}\sqrt{m_2}} & \frac{K_2+K_3}{m_2} & \frac{-K_3}{\sqrt{m_2}\sqrt{m_3}} & \dots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \frac{-K_{n-1}}{\sqrt{m_{n-2}}\sqrt{m_{n-1}}} & \frac{K_{n-1}+K_n}{m_{n-1}} & \frac{-K_n}{\sqrt{m_{n-1}}\sqrt{m_n}} \\ 0 & 0 & \dots & \frac{-K_n}{\sqrt{m_{n-1}}\sqrt{m_n}} & \frac{K_n}{m_n} \end{bmatrix}$$

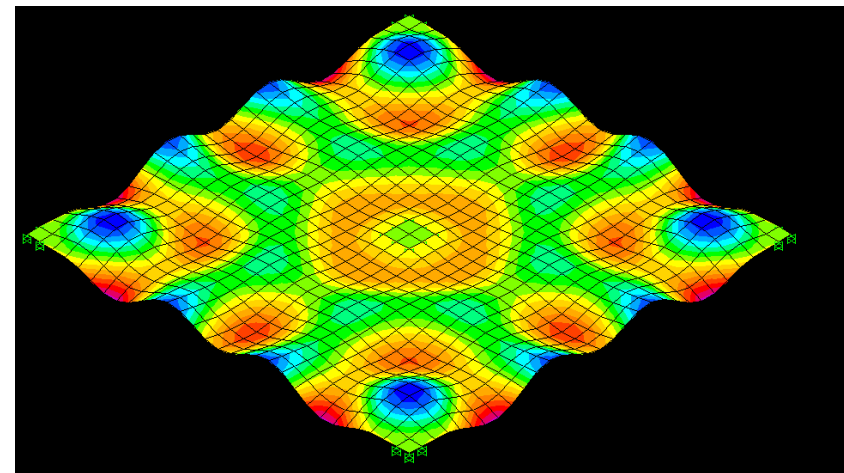
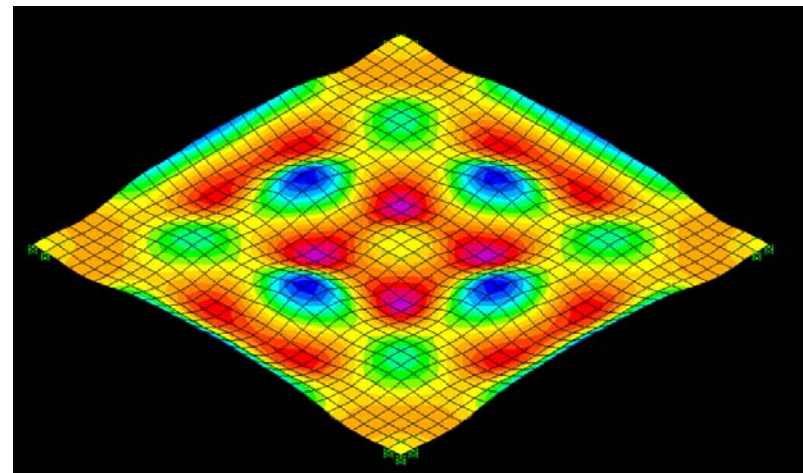
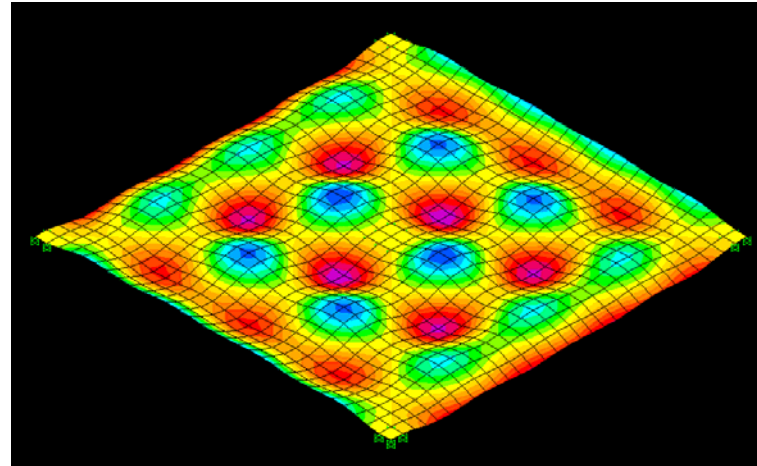
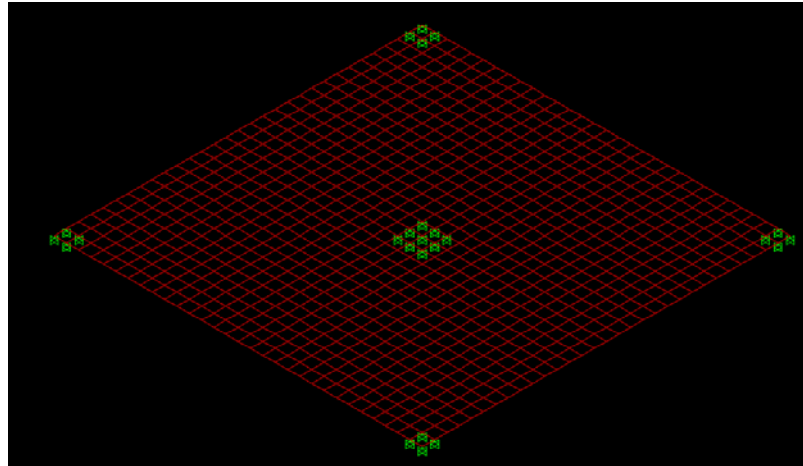

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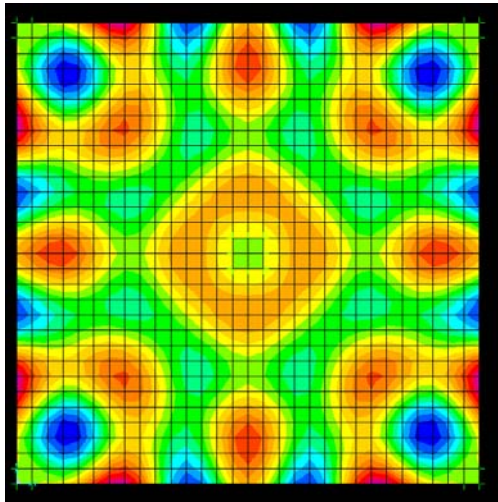
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Mode Shape Analysis of Shell Element (High Freq mode)

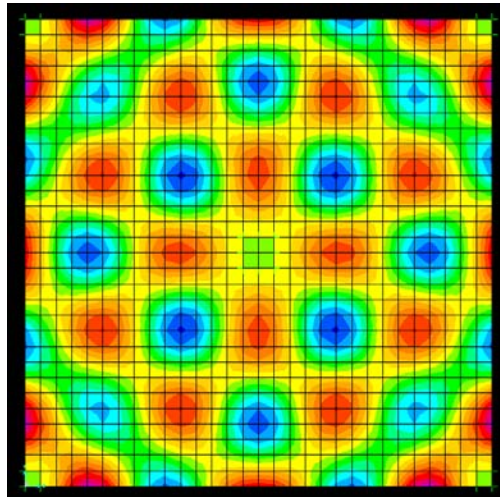


Lec-04 Modal Analysis of MDOF system

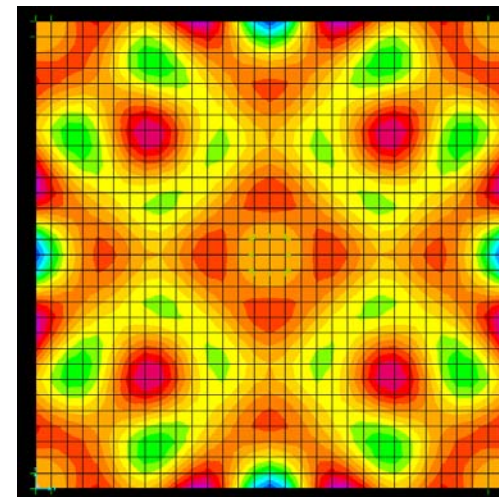
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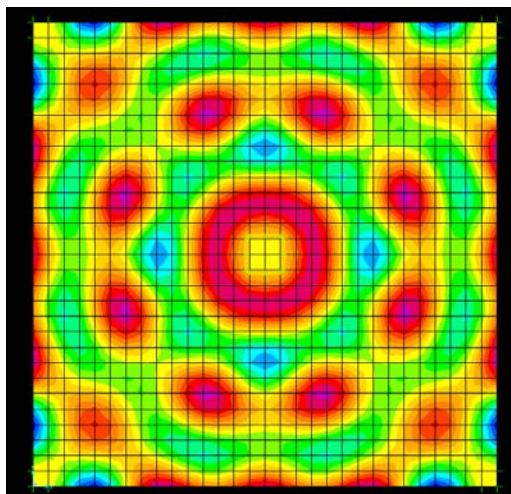
Mode 59 - Period 0.01466



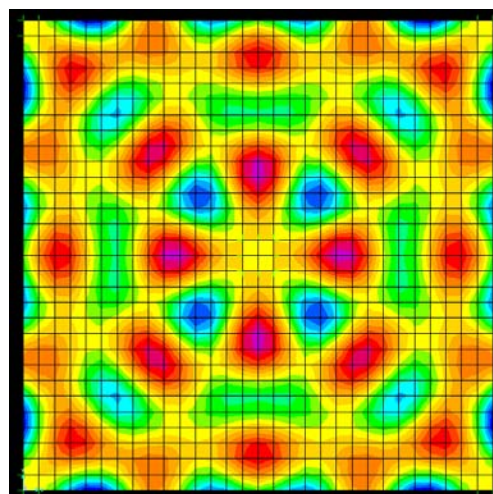
Mode 62 - Period 0.01365



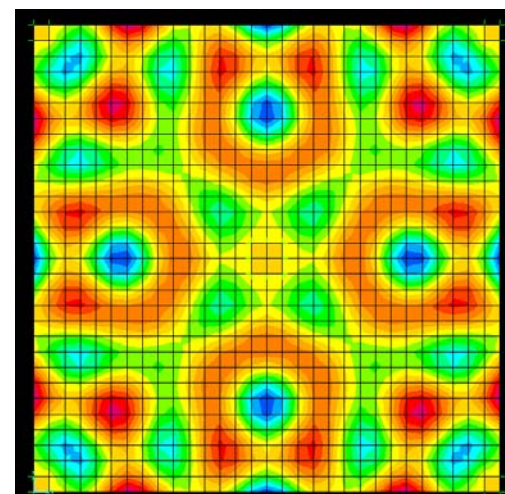
Mode 68 - Period 0.01241



Mode 84 - Period 0.01003



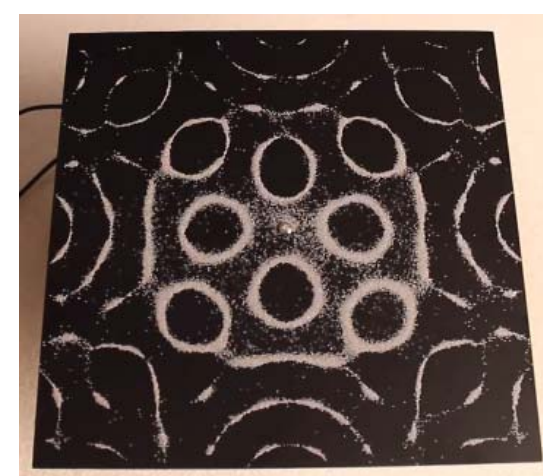
Mode 85 - Period 0.00980



Mode 93 - Period 0.00917

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Lec-02 Vibration of SDOF System

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