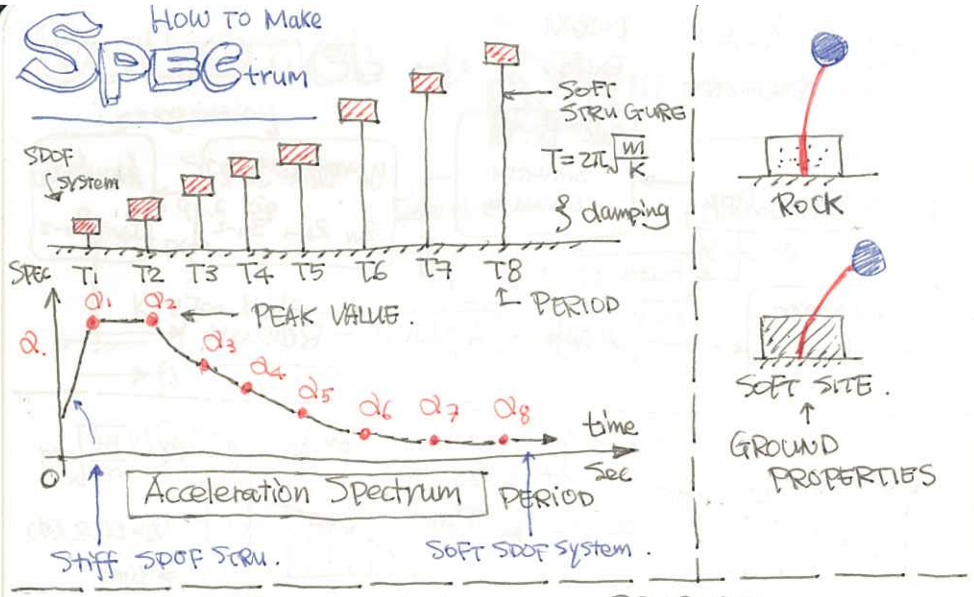
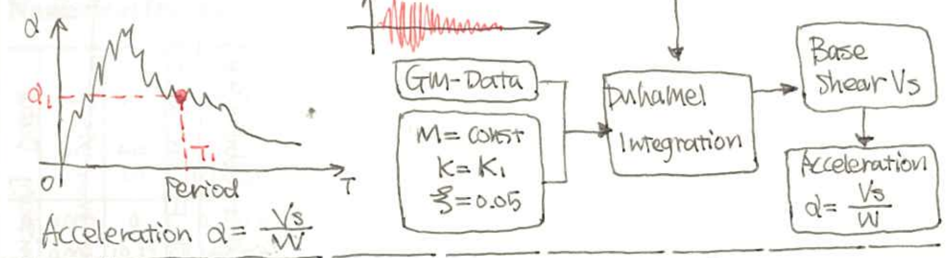
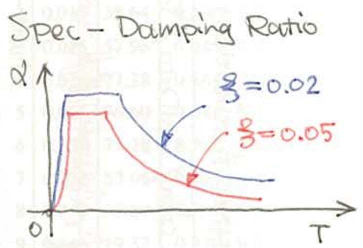
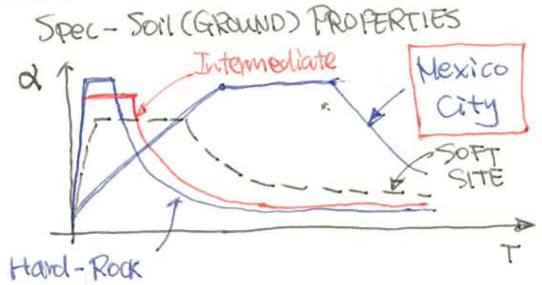
How To Make SPECtrum

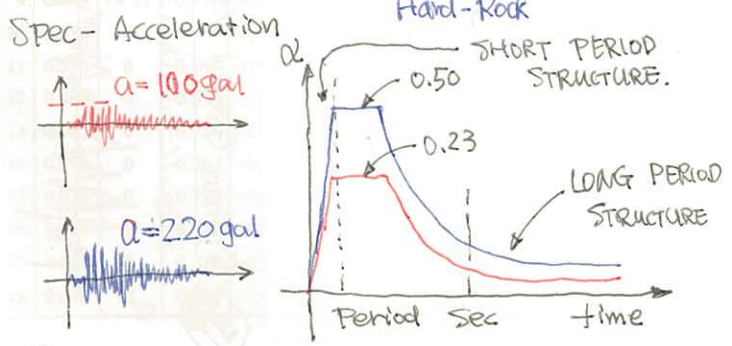


Make Spectrum From GM Data.



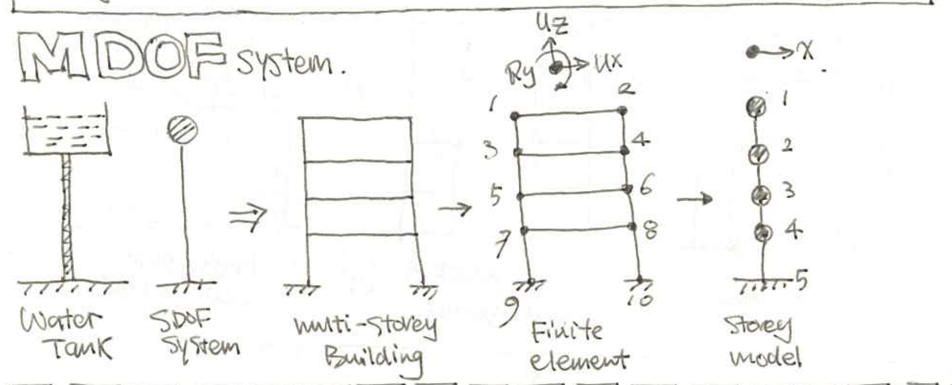






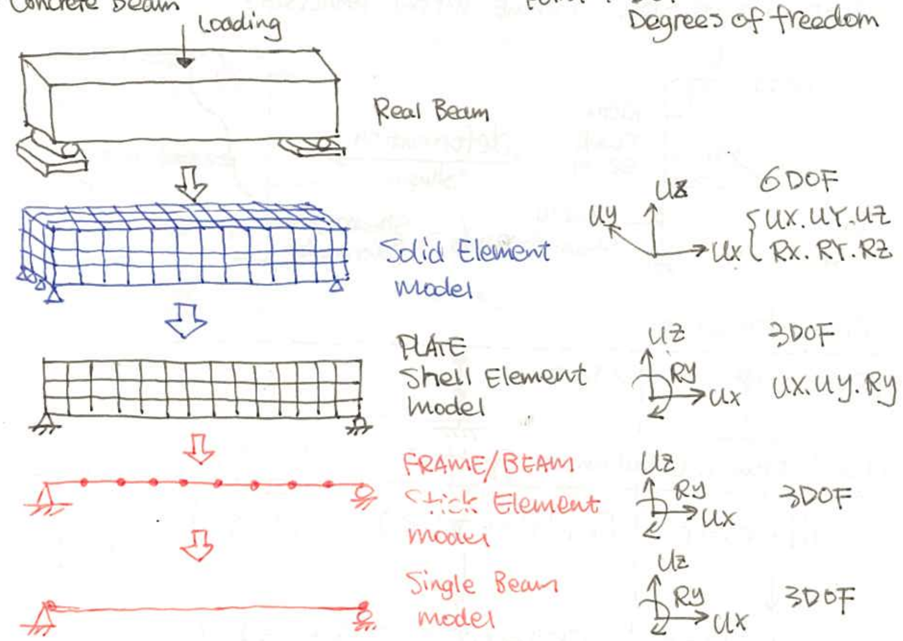
NOTE:
Duhamel Integration is not exact solution for Computing Reponse of Any SDOF system vibration. But It work! [Numerical Solution]

MDOF system.

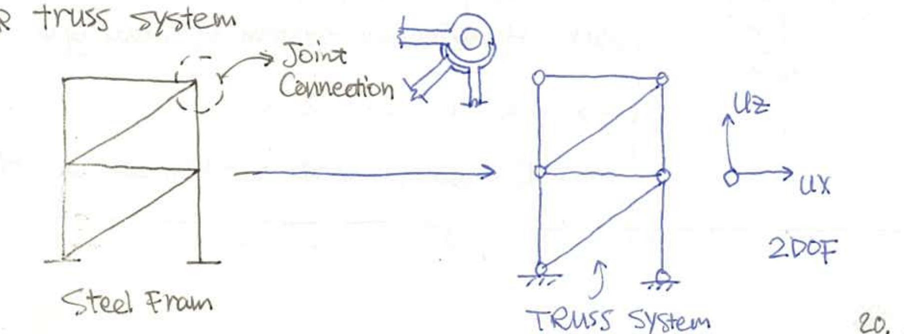


For Concrete Beam

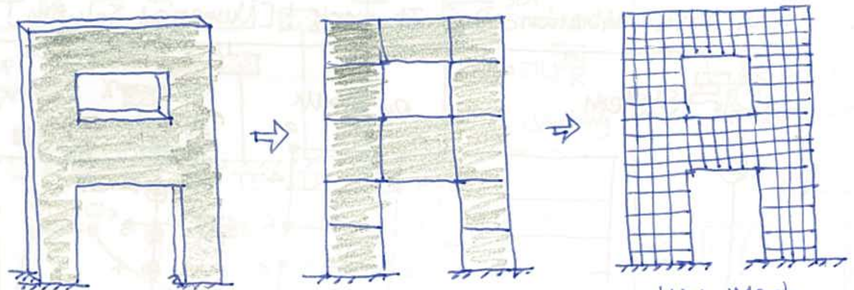
Point. DOF Degrees of freedom



For truss system

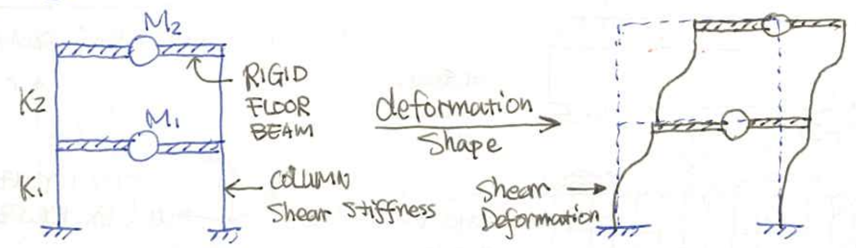


For Shear Wall STRUCTURE



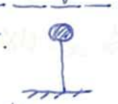
Wall Element — Low Mesh Wall model — High Mesh Wall model.

Typical Two Storey Frame Modal Analysis : (SHEAR FRAME)



Basic Equation

SDOF.

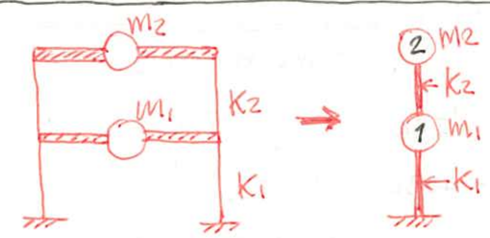


2 DOF

Free Vibration (Undamped System)

SDOF	2 DOF
$M\ddot{u}+Ku=0$	$[M]\{\ddot{u}\}+[K]\cdot\{u\}=\{0\}$
$\downarrow$	$\downarrow$
$K-\omega^2\cdot M=0$	$([K]-\omega^2\cdot[M])\{\phi\}=\{0\}$
$\omega=\sqrt{\frac{K}{\omega}}$.	Math : Homogeneous system of linear equations.
	$\hookrightarrow \left\|[K]-\omega^2\cdot[M]\right\|=0$
	Math : Nesessary Condition For Non-Zero Solution.

Two DOF SYSTEM. Modal Analysis.



$M_1 = 60$ ton

$M_2 = 50$ ton

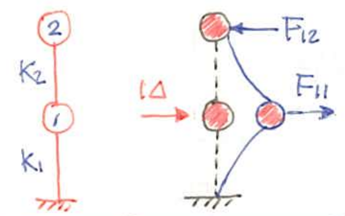
$K_1 = 5\times10^4$ kN/m (N/mm)

$K_2 = 3\times10^4$ kN/m (N/mm)

Stiffness matrix [K]

$$[K]=\begin{bmatrix}K_{11} & K_{12}\\ K_{21} & K_{22}\end{bmatrix}$$

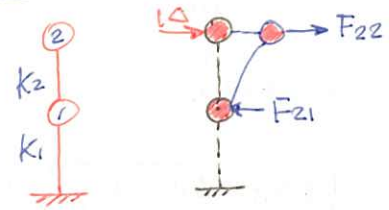
K_{ij}. Apply Unit Disp on Point i.
Get Reaction Force From Point j



$F_{11} = K_1 + K_2 = K_{11}$

$F_{12} = -K_2 = K_{12}$

← Deformation method.



$F_{22} = K_2 = K_{22}$

$F_{21} = -K_2 = K_{21}$

$$[K]=\begin{bmatrix}K_1+K_2 & -K_2\\ -K_2 & K_2\end{bmatrix}=\begin{bmatrix}8 & -3\\ -3 & 3\end{bmatrix}\times10^4\ (\mathrm{kN/m})$$

$$[M]=\begin{bmatrix}M_{11} & \\ & M_{22}\end{bmatrix}=\begin{bmatrix}60 & \\ & 50\end{bmatrix}\ (\mathrm{ton})$$

$$\left|[K]-\omega^2\cdot[M]\right|=\begin{vmatrix}8\times10^4-60\omega^2 & -3\times10^4\\ -3\times10^4 & 3\times10^4-50\omega^2\end{vmatrix}\qquad(\omega^2=A)$$

$$=3A^2-5.8\times10^3A+1.5\times10^6=0$$

2DOF Modal Analysis.

Stru Period.

$$3A^2 - 5.8\times10^3 A + 1.5\times10^6 = 0$$

$$\rightarrow \begin{cases} A_1 = \omega_1^2 = 307.6 & \omega_1 = 17.54 & T_1 = 0.358 \text{ (sec)} \\ A_2 = \omega_2^2 = 1625.8 & \omega_2 = 40.32 & T_2 = 0.156 \text{ (sec)} \end{cases}$$

$$([K] - \omega^2[M])\{\phi\} = \{0\} \qquad \{\phi\} \rightarrow \text{mode shape.}$$

$\omega_1 = 17.54 \quad \omega_1^2 = 307.6.$

$$\left[\begin{pmatrix} 8\times10^4 & -3\times10^4 \\ -3\times10^4 & 3\times10^4 \end{pmatrix} - \omega_1^2 \cdot \begin{pmatrix} 60 & 0 \\ 0 & 50 \end{pmatrix}\right] \cdot \{\phi\} = \{0\}$$

$$\Rightarrow \begin{bmatrix} 8\times10^4 - 307.6\times60 & -3\times10^4 \\ -3\times10^4 & 3\times10^4 - 50\times307.6 \end{bmatrix} \cdot \{\phi\} = \{0\}$$

$$\Rightarrow \begin{bmatrix} 6.15 & -3 \\ -3 & 1.46 \end{bmatrix} \times 10^4 \cdot \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \begin{cases} 6.15x_1 - 3x_2 = 0 \\ -3x_1 + 1.46x_2 = 0 \end{cases} \Rightarrow x_1 = \frac{3x_2}{6.15} = 0.488x_2$$

$$\Rightarrow \phi_1 = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0.488x_2 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0.488 \\ 1.00 \end{Bmatrix} \quad \boxed{\text{First Mode Shape}}$$

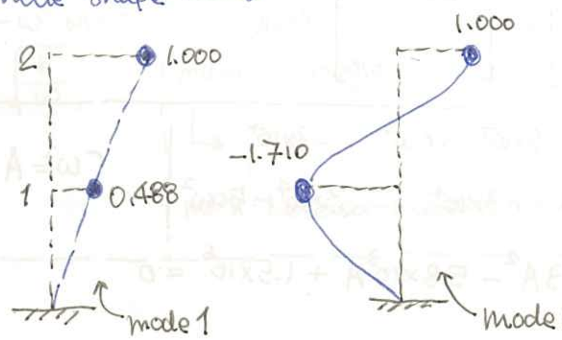
Same procedure

$$\phi_2 = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} -1.710 \\ 1.000 \end{Bmatrix}$$

$ax^2 + bx + c = 0$

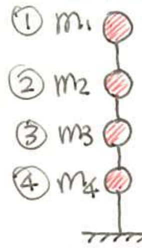
Solution $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Plot mode shape $\longrightarrow$



Mode Shape

4DOF MDOF SYSTEM MODAL ANALYSIS



$K_1 = 1\times800$ KN/m $\quad m_1 = 1$ ton

$K_2 = 2\times800$ KN/m $\quad m_2 = 2$ ton

$K_3 = 3\times800$ KN/m $\quad m_3 = 2$ ton

$K_4 = 4\times800$ KN/m $\quad m_4 = 3$ ton.

$$[M] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix} \qquad [K] = \begin{bmatrix} K_1 & -K_1 & & \\ -K_1 & K_1+K_2 & -K_2 & \\ & -K_2 & K_2+K_3 & -K_3 \\ & & -K_3 & K_3+K_4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & & \\ -1 & 3 & -2 & \\ & -2 & 5 & -3 \\ & & -3 & 7 \end{bmatrix} \times 800 \text{ (KN/m)}$$

$$\left|[K] - \omega^2[M]\right| = 0$$

$$= \begin{vmatrix} 800-\omega^2 & -800 & 0 & 0 \\ -800 & 2400-\omega^2 & -1600 & 0 \\ 0 & -1600 & 4000-\omega^2 & -2400 \\ 0 & 0 & -2400 & 5600-\omega^2 \end{vmatrix}$$

Determinant Calculation.

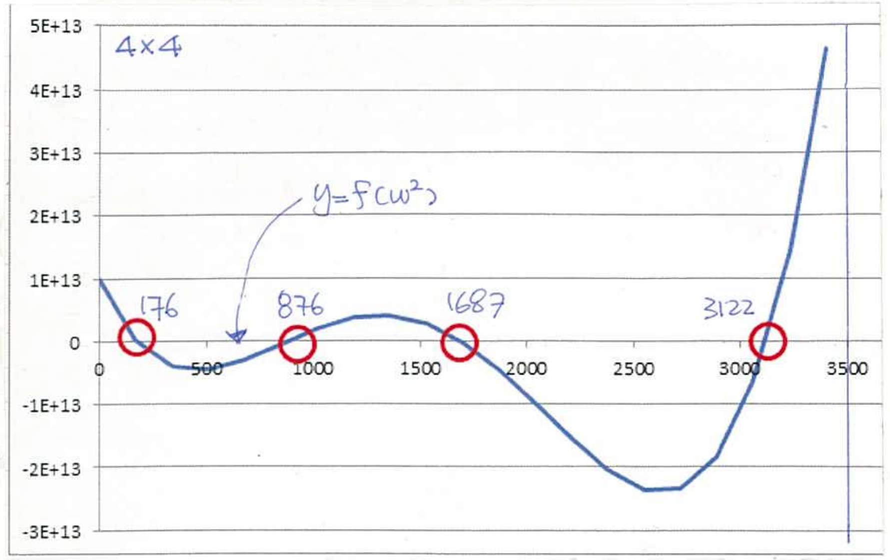
$$\begin{vmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{vmatrix} = a\begin{vmatrix} f & g & h \\ j & k & l \\ n & o & p \end{vmatrix} - b\begin{vmatrix} e & g & h \\ i & k & l \\ m & o & p \end{vmatrix} + c\begin{vmatrix} e & f & h \\ i & j & l \\ m & n & p \end{vmatrix} - d\begin{vmatrix} e & f & g \\ i & j & k \\ m & n & o \end{vmatrix}$$

$$= (800-\omega^2)\cdot\begin{vmatrix} 2400-2\omega^2 & -1600 & 0 \\ -1600 & 4000-2\omega^2 & -2400 \\ 0 & -2400 & 5600-3\omega^2 \end{vmatrix} - (-800)\begin{vmatrix} -800 & -1600 & 0 \\ 0 & 4000-2\omega^2 & -2400 \\ 0 & -2400 & 5600-3\omega^2 \end{vmatrix}$$

$$= (800-\omega^2)\cdot[(2400-2\omega^2)(4000-2\omega^2)(5600-3\omega^2) - (-2400)(-2400)(2400-2\omega^2)$$
$$-(-1600)(-1600)\cdot(5600-3\omega^2)] - (-800)\cdot[(-800)\cdot(4000-2\omega^2)(5600-3\omega^2)$$
$$-(-2400)(-2400)(-800)] = y = f(\omega^2) = 0.$$

$\longrightarrow$ EXCEL PROGRAM.

USE EXCEL PROGRAM TO SOLVE $\boxed{\text{Determinant}}$



$x=\omega^2$

$y=f(\omega^2)$ excel form.

step	x	y
0	0	9.8304E+12
1	170	2.56339E+11
2	340	-3.96523E+12
3	510	-4.54876E+12
4	680	-2.96814E+12
5	850	-4.56725E+11
6	1020	1.99265E+12
7	1190	3.62772E+12
8	1360	3.93675E+12
9	1530	2.64852E+12
10	1700	-2.676E+11
11	1870	-4.60174E+12
12	2040	-9.90348E+12
13	2210	-1.54819E+13
14	2380	-2.04053E+13
15	2550	-2.35019E+13
16	2720	-2.3359E+13
17	2890	-1.83234E+13
18	3060	-6.50158E+12
19	3230	1.42408E+13
20	3400	4.62784E+13

$$\begin{bmatrix} x_1=176 \\ x_2=876 \\ x_3=1687 \\ x_4=3122 \end{bmatrix} \rightarrow \begin{bmatrix} \omega_1=13.2 \\ \omega_2=29.65 \\ \omega_3=41.07 \\ \omega_4=55.87 \end{bmatrix}$$

$$\begin{bmatrix} T_1=0.474\ s \\ T_2=0.212\ s \\ T_3=0.153\ s \\ T_4=0.112\ s \end{bmatrix}$$ Period

MODE SHAPE OF 4DOF SYSTEM

FOR EXAMPLE:

Mode shape 2 ($T_2=0.212$ Sec) $\omega^2=876$ Divide matrix

$$D(\omega^2)=[K]-\omega^2[M]=\begin{bmatrix} -76 & -800 & 0 & 0 \\ -800 & 648 & -1600 & 0 \\ 0 & -1600 & 2248 & -2400 \\ 0 & 0 & -2400 & 2972 \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix}$$

$$D_{11}=[-76] \quad D_{12}=\begin{bmatrix} -800 \\ 0 \\ 0 \end{bmatrix} \quad D_{22}=\begin{bmatrix} 648 & -1600 & 0 \\ -1600 & 2248 & -2400 \\ 0 & -2400 & 2972 \end{bmatrix}$$

$$D_{22}^{-1}=\frac{1}{219108616}\times\begin{bmatrix} -28783 & -148600 & -120000 \\ -148600 & -60183 & -48600 \\ -120000 & -48600 & 34478 \end{bmatrix}$$ math Inversion

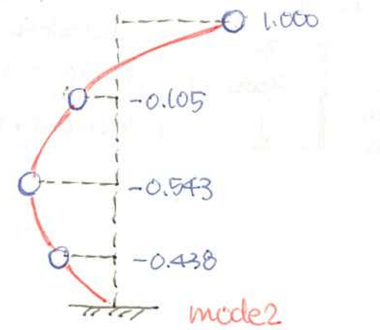
inv[matrix] WolframAlpha®

$$\phi_2=\begin{Bmatrix} 1.000 \\ \phi_{2r} \end{Bmatrix}$$

$$\phi_{2r}=-D_{22}^{-1}\cdot D_{12}=\frac{-1}{219108616}\times\begin{bmatrix} -28783 & -148600 & -120000 \\ -148600 & -60183 & -48600 \\ -120000 & -48600 & 34478 \end{bmatrix}\cdot\begin{bmatrix} -800 \\ 0 \\ 0 \end{bmatrix}$$

$$=\begin{bmatrix} -0.105 \\ -0.543 \\ -0.438 \end{bmatrix}$$

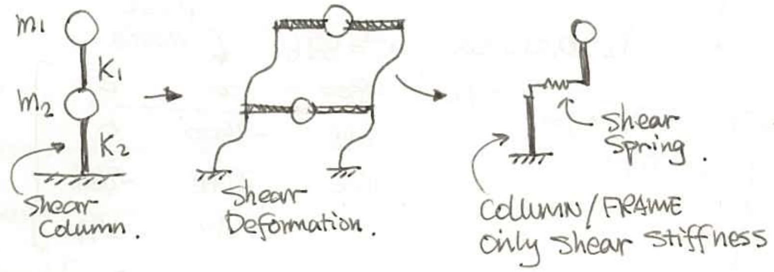
$$\phi_2=\begin{bmatrix} 1.000 \\ \phi_{2r} \end{bmatrix}=\begin{bmatrix} 1.000 \\ -0.105 \\ -0.543 \\ -0.438 \end{bmatrix} \xrightarrow{\text{mode shape}}$$



mode2

$T_2=0.212$ (Sec)

How to model shear column in ETABS Program



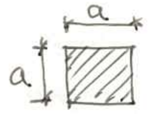
Shear Stiffness $K = \frac{GA_s}{L} \Rightarrow a = \sqrt{A} = \sqrt{A_s \cdot \frac{1}{0.8333}} = \sqrt{\frac{kL}{G} \times \frac{1}{0.8333}}$

If $K = 800$ (kN/m) $= 800$ (N/mm)

$L = 3000$ (mm)

$G = 10416$ (MPa) ← shear modulus.

$a = \sqrt{\frac{kL}{0.8333 \cdot G}} = 16.628$ (mm)

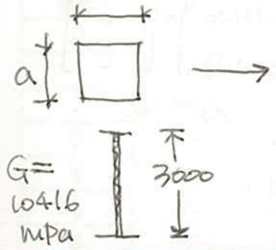


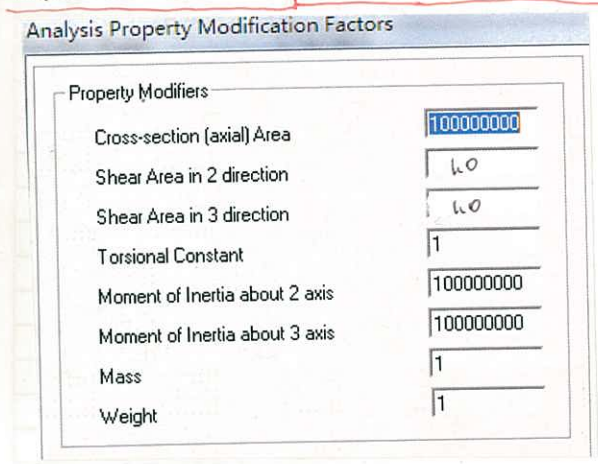
$A = a^2$

$A_s = 0.8333\,A$.

A_s: Shear Area.

FRAME SECTION	FRAME SECTION PROPETIES MODIFIED FACTOR		
$a = 16.628$	Axial Area A.	10^8	Big.
	Shear Area along Axis 3	1.0	
	Shear Area along Axis 2	1.0	
	Torsional Constant T	1.0	
G= 10416 MPa	Moment of Inertia about Axis 2	10^8	Big
	Moment of Inertia about Axis 3	10^8	Big




Analysis Property Modification Factors

Property Modifiers

Cross-section (axial) Area	100000000
Shear Area in 2 direction	1.0
Shear Area in 3 direction	1.0
Torsional Constant	1
Moment of Inertia about 2 axis	100000000
Moment of Inertia about 3 axis	100000000
Mass	1
Weight	1

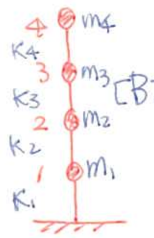
Numerical method for modal analysis.

$[M]\cdot\{\ddot{x}\} + [K]\cdot\{x\} = \{0\}$ Assume $\{z\} = [\sqrt{M}]\cdot\{x\}$

$\rightarrow \left|[K] - \omega^2[M]\right| = 0$ $[B] = [\sqrt{M}]^{-1}\cdot[K]\cdot[\sqrt{M}]^{-1}$

$\rightarrow \left|[B]\right| = 0 \rightarrow \{z\} \longrightarrow \{x\} = [\sqrt{M}]^{-1}\cdot\{z\}$

Storey Stiffness model



$$[B] = \begin{bmatrix} \frac{K_1+K_2}{m_1} & \frac{-K_2}{\sqrt{m_1}\sqrt{m_2}} & 0 & 0 \\ \frac{-K_2}{\sqrt{m_1}\sqrt{m_2}} & \frac{K_2+K_3}{m_2} & \frac{-K_3}{\sqrt{m_2}\sqrt{m_3}} & 0 \\ 0 & \frac{-K_3}{\sqrt{m_2}\sqrt{m_3}} & \frac{K_3+K_4}{m_3} & \frac{-K_4}{\sqrt{m_3}\sqrt{m_4}} \\ 0 & 0 & \frac{-K_4}{\sqrt{m_3}\sqrt{m_4}} & \frac{K_4}{m_4} \end{bmatrix}$$

← Eigen Value of matrix [B]

$$[B] = \begin{bmatrix} \frac{K_1+K_2}{m_1} & \frac{-K_2}{\sqrt{m_1}\sqrt{m_2}} & 0 & \cdots & 0 \\ \frac{-K_2}{\sqrt{m_1}\sqrt{m_2}} & \frac{K_2+K_3}{m_2} & \frac{-K_3}{\sqrt{m_2}\sqrt{m_3}} & \cdots & 0 \\ \vdots & \vdots & \ddots & & \\ \vdots & \vdots & \frac{-K_{n-1}}{\sqrt{m_{n-2}}\sqrt{m_{n-1}}} & \frac{K_{n-1}+K_n}{m_{n-1}} & \frac{-K_n}{\sqrt{m_{n-1}}\sqrt{m_n}} \\ 0 & 0 & & \frac{-K_n}{\sqrt{m_{n-1}}\sqrt{m_n}} & \frac{K_n}{m_n} \end{bmatrix}$$

Lazarus Delphi Program.

Numerical Method

Jacobi Method → Jacobi[B.n.d.V.nrot]

B → Matrix B.	n → degree Num.	d → eigen Value.
V → Vector Matrix	nrot → Rotation Number	

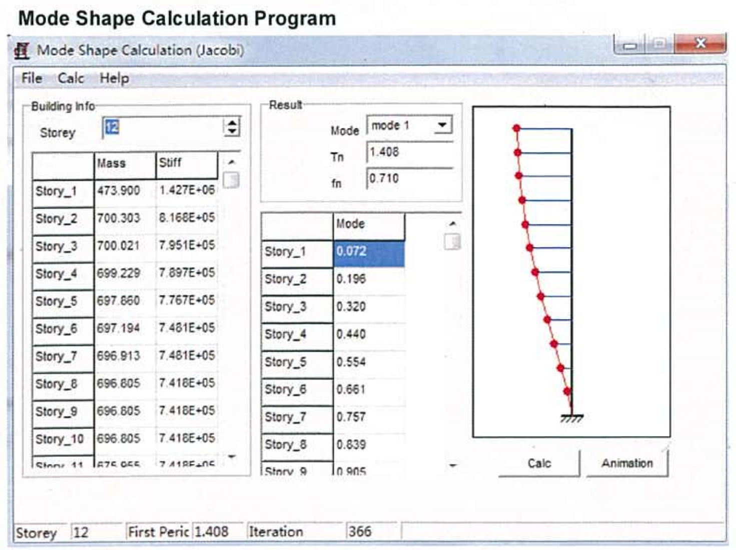
← Iteration

Form [B] matrix Pascal:

```
For i = 2 to n-1 do
begin
    B[i, i]= (K[i]+K[i+1])/m[i] ;
    B[i. i-1]=(-K[i])/SQRT(m[i-1])/SQRT(m[i]);
    B[i, i+1]= (-K[i+1])/SQRT(m[i+1])/SQRT (m[i]);
end;
```

PROGRAM by Delphi/Lazarus

Modal Analysis For Storey-stiffness model:



Program method.

1. FORM matrix [M]
2. FORM matrix [K]
3. FORM matrix [B]
4. JCOBI Method
5. Eigen Value
6. Period Frequency

FORM matrix $[\sqrt{M}]$

Eigen Vector {z}

Gauss Method $[\sqrt{M}]\cdot\{x\}=\{z\}$

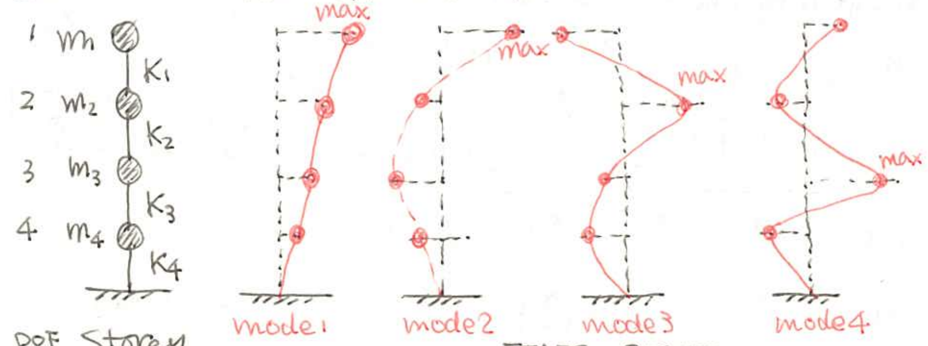
7. mode shape {x}

Main Equation

$$[B]=[\sqrt{M}]^{-1}[K][\sqrt{M}]^{-1}$$

$$[z]=[\sqrt{M}]\cdot[x]$$

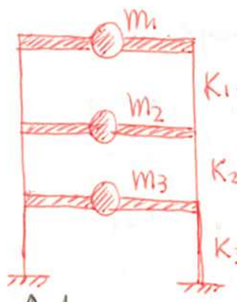
Solution For 4DoF Typical Example



ETABS RESULT.

DOF	Storey				
①	4	1.000	1.000	−0.9034	0.1542
②	3	0.7767	−0.1014	1.000	−0.4487
③	2	0.4937	−0.5426	−0.1581	1.000
④	1	0.2326	−0.4398	−0.7083	−0.6368

VIBRATION CALCULATION OF MDOF System (Important)



$K_1 = 60$ kN/m $m_1 = 1.0$ ton

$K_2 = 120$ kN/m $m_2 = 1.5$ ton

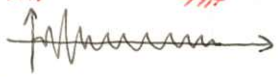
$K_3 = 180$ kN/m $m_3 = 2.0$ ton.

Information Need:

① Duhamel Integration

② Mode shape Calculation

③ BIG EXCEL Table



GROUND MOTION: EL-Centro Wave $a_{max} = 100gal$

Important: MDOF EQUATION: $[M]\ddot{u} + [C]\cdot\dot{u} + [K]\cdot u = P(t)$

Transformation of Coordinates: $u = \Phi \cdot y$.

$\Rightarrow M\Phi\ddot{y} + C\Phi\dot{y} + K\Phi y = P(t)$

Premultiply by $\Phi^T \Longrightarrow \Phi^T M\Phi\ddot{y} + \Phi^T C\Phi\dot{y} + \Phi^T K\Phi y = \Phi^T P(t)$.