CIVL7008 Seismic Analysis for Building Structures

Lec-03 Forced Vibration of SDOF System

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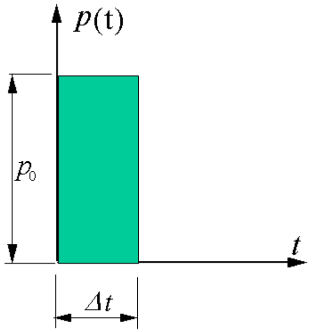
Force Vibration: Rectangle Impulse Excitation



$$I = p_0 \Delta t$$

$$m\ddot{v} + c\dot{v} + kv = 0$$

$$v(t) = e^{-\xi\omega t}\left[\frac{\dot{v}_0 + v_0\xi\omega}{\omega_d}\sin\omega_d t + v_0\cos\omega_d t\right]$$

$v_0 = 0$, $\quad \dot{v}_0 = \dfrac{p_0}{m}\Delta t$

$$v(t) = e^{-\xi\omega t}\frac{p_0\Delta t}{m\omega_d}\sin\omega_d t$$

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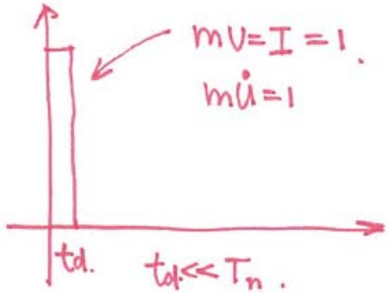
Rectangular Impulse



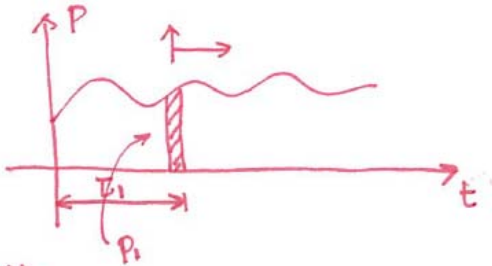
$$\begin{cases} \dot{u}(0) = \frac{1}{m} \\ u(0) = 0 \end{cases}$$ initial Condition.

$$u(t) = u_0 \cdot \cos\omega_n t + \left(\frac{\dot{u}_0}{\omega_n}\right) \sin\omega_n t$$

$$u(t) = 0 + \left(\frac{1}{m\omega_n}\right) \cdot \sin\omega_n t.$$ For Undamped system.

Damped System.

$$u(t) = \exp(-\xi\omega_n t) \cdot \frac{1}{m\omega_d} \cdot \sin\omega_d t.$$



Unit:

$$\begin{cases} u(t) = \exp[-\xi\omega_n(t-\tau_1)] \cdot \sin\omega_d(t-\tau_1) \cdot \frac{I}{m\omega_d} \\ I = P_1 \cdot d\tau \end{cases}$$

Particular Solution

$$u_p = \int_0^t \left\{ \frac{P(\tau)}{m\omega_d} \cdot \exp[-\xi\omega(t-\tau)] \cdot \sin\omega_d(t-\tau) \right\} d\tau$$

↑ Duhamel Integration.

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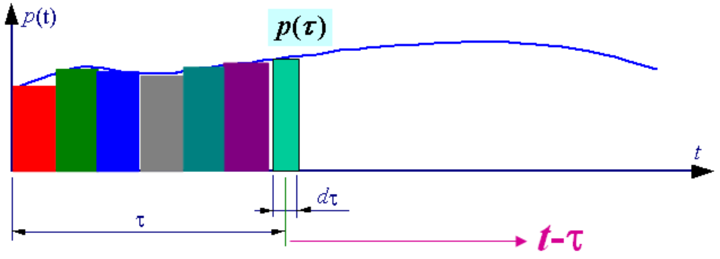

From $v(t) = e^{-\xi\omega t}\frac{p_0\Delta t}{m\omega_d}\sin\omega_d t$

$t \to t-\tau$

$p_0 \to p(\tau)$

$\Delta t \to d\tau$

$$\Rightarrow \mathrm{d}v(t) = \frac{p(\tau)\mathrm{d}\tau}{m\omega_\mathrm{d}}e^{-\xi\omega(t-\tau)}\sin\omega_\mathrm{d}(t-\tau)$$

$$v(t) = \int_0^t \frac{p(\tau)}{m\omega_\mathrm{d}}e^{-\xi\omega(t-\tau)}\sin\omega_\mathrm{d}(t-\tau)\mathrm{d}\tau$$

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Damped – system.

$$u_p = \int_0^t \left[\frac{p(\tau)}{m\omega_d} \cdot \exp[-\xi\omega(t-\tau)] \cdot \sin\omega_d(t-\tau) \right] d\tau$$

$$= \int_0^t \left[\frac{p(\tau)}{m\omega_d} \cdot \frac{\exp(\xi\omega\tau)}{\exp(\xi\omega t)} \cdot [\sin\omega_d t * \cos\omega_d\tau - \cos\omega_d t * \sin\omega_d\tau] \right] d\tau$$

$$= \frac{1}{m\omega_d} \int_0^t p(\tau) \cdot \frac{\exp(\xi\omega\tau)}{\exp(\xi\omega t)} \cdot \cos\omega_d\tau \cdot \underline{\underline{\sin\omega_d t}}\, d\tau - \frac{1}{m\omega_d} \int_0^t p(\tau) \cdot \frac{\exp(\xi\omega\tau)}{\exp(\xi\omega t)} \cdot \sin\omega_d\tau \cdot \underline{\underline{\cos\omega_d t}}\, d\tau$$

$$= \sin\omega_d t \cdot \frac{1}{m\omega_d} \cdot \int_0^t p(\tau) \cdot \frac{\exp(\xi\omega\tau)}{\exp(\xi\omega t)} \cdot \cos\omega_d\tau \cdot d\tau - \cos\omega_d t \cdot \frac{1}{m\omega_d} \int_0^t p(\tau) \cdot \frac{\exp(\xi\omega\tau)}{\exp(\xi\omega t)} \cdot \sin\omega_d\tau \cdot d\tau$$

$$= \sin\omega_d t \cdot A - \cos\omega_d t \cdot B.$$

$$A = \frac{1}{m\omega_d} \cdot \int_0^t p(\tau) \cdot \frac{\exp(\xi\omega\tau)}{\exp(\xi\omega t)} \cdot \cos\omega_d\tau \cdot d\tau$$

$$y = p(\tau) \cdot \cos\omega\tau.$$

$$A = \frac{1}{m\omega_d} \cdot \int_0^t \frac{\exp(\xi\omega\tau)}{\exp(\xi\omega t)} \cdot y \cdot d\tau$$

Simple summation:

$$\bar{A}_N = \bar{A}_{N-1} \cdot \exp(-\xi\omega\Delta\tau) + \frac{\Delta\tau}{m\omega} \cdot [y_{N-1}] \cdot \exp(-\xi\Delta\tau).$$

Simpons rule:

$$\bar{A}_N = \bar{A}_{N-2} \cdot \exp(-2\xi\omega\Delta\tau) + \frac{\Delta\tau}{3m\omega_d} \cdot [y_{N-2} \cdot \exp(-\xi\Delta\tau \cdot 2)$$

$$+ 4y_{N-1} \cdot \exp(-\xi\omega\Delta\tau) + y_N].$$

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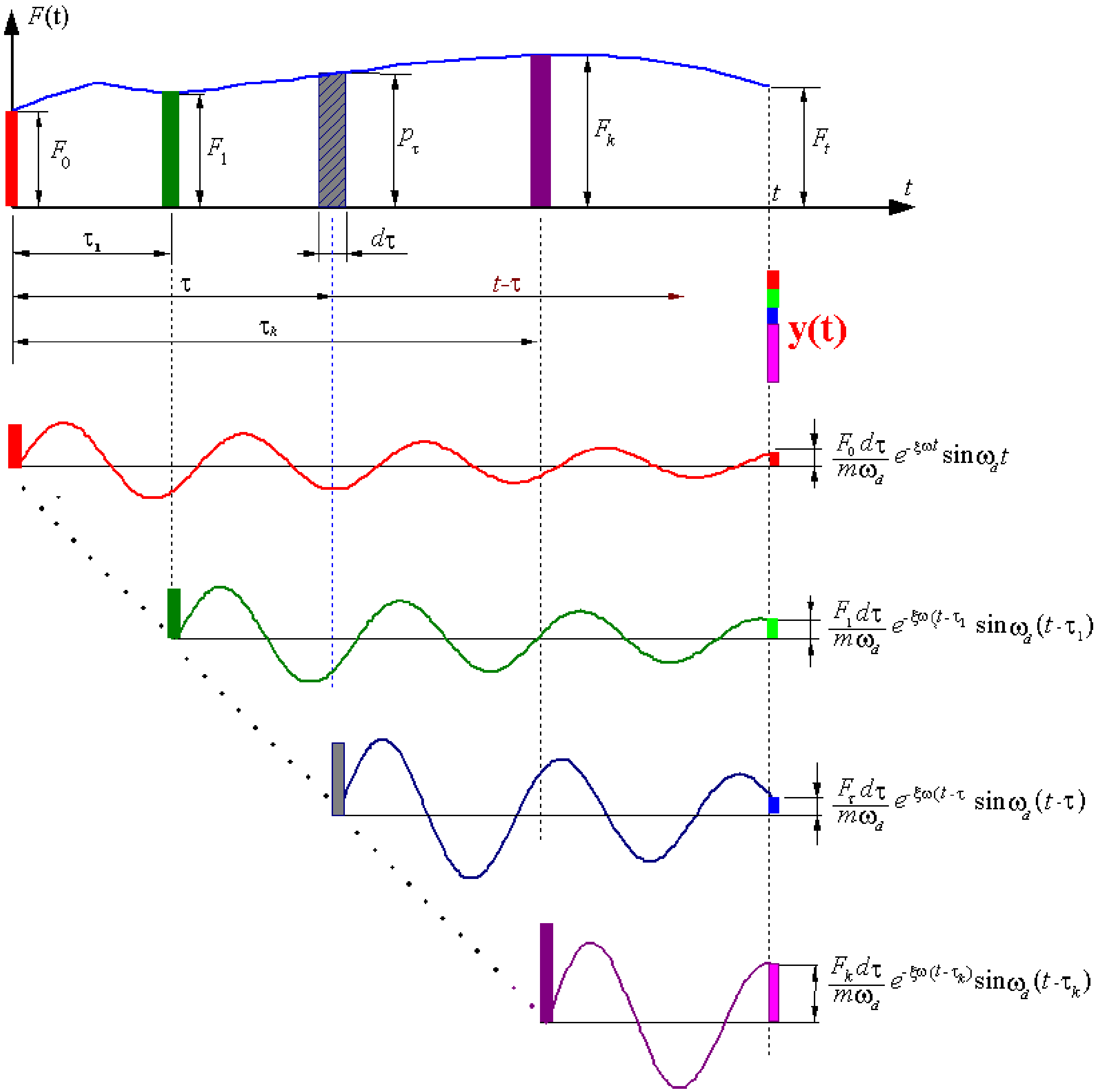


Lec-03 Forced Vibration of SDOF System

Excel Table for Duhamel Integration

$$v(t) = A(t)\ \sin\omega_D t - B(t)\ \cos\omega_D t$$

$$A(t) \equiv \frac{1}{m\,\omega_D} \int_0^t p(\tau)\ \frac{\exp(\xi\omega\tau)}{\exp(\xi\omega t)}\ \cos\omega_D\tau\ d\tau$$

$$B(t) \equiv \frac{1}{m\,\omega_D} \int_0^t p(\tau)\ \frac{\exp(\xi\omega\tau)}{\exp(\xi\omega t)}\ \sin\omega_D\tau\ d\tau$$

Simple summation:

$$A_N \doteq A_{N-1}\ \exp(-\xi\,\omega\,\triangle\tau) + \tfrac{\triangle\tau}{m\,\omega_D}\ y_{N-1}\ \exp(-\xi\,\omega\,\triangle\tau)$$

$$N = 1, 2, 3, \cdots \quad \text{(6-17a)}$$

Trapezoidal rule:

$$A_N \doteq A_{N-1}\ \exp(-\xi\,\omega\,\triangle\tau) + \tfrac{\triangle\tau}{2\,m\,\omega_D}\left[y_{N-1}\ \exp(-\xi\,\omega\,\triangle\tau) + y_N\right]$$

$$N = 1, 2, 3, \cdots \quad \text{(6-17b)}$$

Simpson's rule:

$$A_N \doteq A_{N-2}\ \exp(-2\,\xi\,\omega\,\triangle\tau) + \tfrac{\triangle\tau}{3\,m\,\omega_D}\left[y_{N-2}\ \exp(-2\,\xi\,\omega\,\triangle\tau) + 4\,y_{N-1}\ \exp(-\xi\,\omega\,\triangle\tau) + y_N\right]$$

$$N = 2, 4, 5, \cdots \quad \text{(6-17c)}$$

Excel Table for Duhamel Integration

TABLE E6-2
Numerical Duhamel integral analysis including damping

N	t_N sec	p_N kips (1)	$\sin 30t_N$ (2)	$\cos 30t_N$ (3)	y_N (1)×(3) kips (4)	y_{N-1} (5)	y_{N-2} (6)	$M_1 \times$ (5) (7)	$M_2 \times$ [(6)+(9)] (8)	$\frac{A_{N-2}}{F}$ (9)	$\frac{A_N}{F}$ (4)+(7)+(8) (10)	y_N (1)×(2) kips (11)	y_{N-1} (12)	y_{N-2} (13)	$M_1 \times$ (12) (14)	$M_2 \times$ [(13)+(16)] (15)	$\frac{B_{N-2}}{F}$ (16)	$\frac{B_N}{F}$ (11)+(14)+(15) (17)	(10)×(2) (18)	(17)×(3) (19)	(18)−(19) (20)	v_N F×(20) ft (21)	f_{S_N} k×(21) kips (22)
0	0.000	0	0	1.000	0	—	—	—	—	—	0	0	—	—	—	—	—	0	0	0	0	0	0
1	0.005	19.32	0.149	0.989	19.1	0	—	—	—	—	—	2.88	0	—	—	—	—	—	—	—	—	—	—
2	0.010	38.64	0.296	0.955	36.9	19.1	0	75.8	0	0	112.7	11.4	2.88	0	11.4	0	0	22.8	33.3	21.8	11.5	0.0002	0.58
3	0.015	57.96	0.435	0.900	52.2	36.9	19.1	—	—	—	—	25.2	11.4	2.88	—	—	—	—	—	—	—	—	—
4	0.020	77.28	0.565	0.825	63.8	52.2	36.9	207.2	147.4	112.7	418.4	43.7	25.2	11.4	100.0	33.7	22.8	177.4	236	146	90	0.0017	4.50
5	0.025	96.60	0.682	0.732	70.7	63.8	52.2	—	—	—	—	65.9	43.7	25.2	—	—	—	—	—	—	—	—	—
6	0.030	77.28	0.783	0.622	48.1	70.7	63.8	280.7	475.0	418.4	803.8	60.5	65.9	43.7	261.6	217.8	177.4	539.9	629	336	293	0.0054	14.65
7	0.035	57.96	0.867	0.498	28.9	48.1	70.7	—	—	—	—	50.3	60.5	65.9	—	—	—	—	—	—	—	—	—
8	0.040	38.64	0.932	0.362	14.0	28.9	48.1	114.7	839.1	803.8	967.8	36.0	50.3	60.5	199.7	591.4	539.9	827.1	902	299	603	0.0112	30.2
9	0.045	19.32	0.976	0.219	4.23	14.0	28.9	—	—	—	—	18.9	36.0	50.3	—	—	—	—	—	—	—	—	—
10	0.050	0	0.997	0.0707	0	4.23	14.0	16.8	967.1	967.8	983.9	0	18.9	36.0	75.0	850.1	827.1	925.1	981	65.4	915	0.0169	45.8
11	0.055	0	0.997	−0.0791	0	0	4.23	—	—	—	—	0	0	18.9	—	—	—	—	—	—	—	—	—
12	0.060	0	0.974	−0.227	0	0	0	0	969.1	983.9	969.1	0	0	0	0	911.2	925.1	911.2	900	−206	1106	0.0205	55.4
13	0.065	0	0.929	−0.370	0	0	0	—	—	—	—	0	0	0	—	—	—	—	—	—	—	—	—
14	0.070	0	0.863	−0.505	0	0	0	0	954.6	969.1	0	0	0	0	0	897.5	911.2	897.5	824	−453	1277	0.0236	63.9
15	0.075	0	0.778	−0.628	0	0	0	—	—	—	—	0	0	0	—	—	—	—	—	—	—	—	—
16	0.080	0	0.675	−0.737	0	0	0	0	940.3	954.6	940.3	0	0	0	0	884.0	897.5	884.0	635	−651.5	1286	0.0238	64.3
17	0.085	0	0.558	−0.830	0	0	0	—	—	—	—	0	0	0	—	—	—	—	—	—	—	—	—
18	0.090	0	0.427	−0.904	0	0	0	0	926.2	940.3	926.2	0	0	0	0	870.7	884.0	870.7	395	−787	1182	0.0219	59.1

$\omega = \sqrt{\frac{kg}{W}} = 30\ rad/sec$ $\Delta\tau = 0.005\ sec$ $M_1 = 4\exp(-\xi\omega\Delta\tau) = 3.97$ $M_2 = \exp(-2\xi\omega\Delta\tau) = 0.985$ $F = \frac{\Delta\tau}{3m\omega} = 1.852 \times 10^{-5}\ ft/kip$ $k = 2700\ kips/ft$

Excel Table for Duhamel Integration

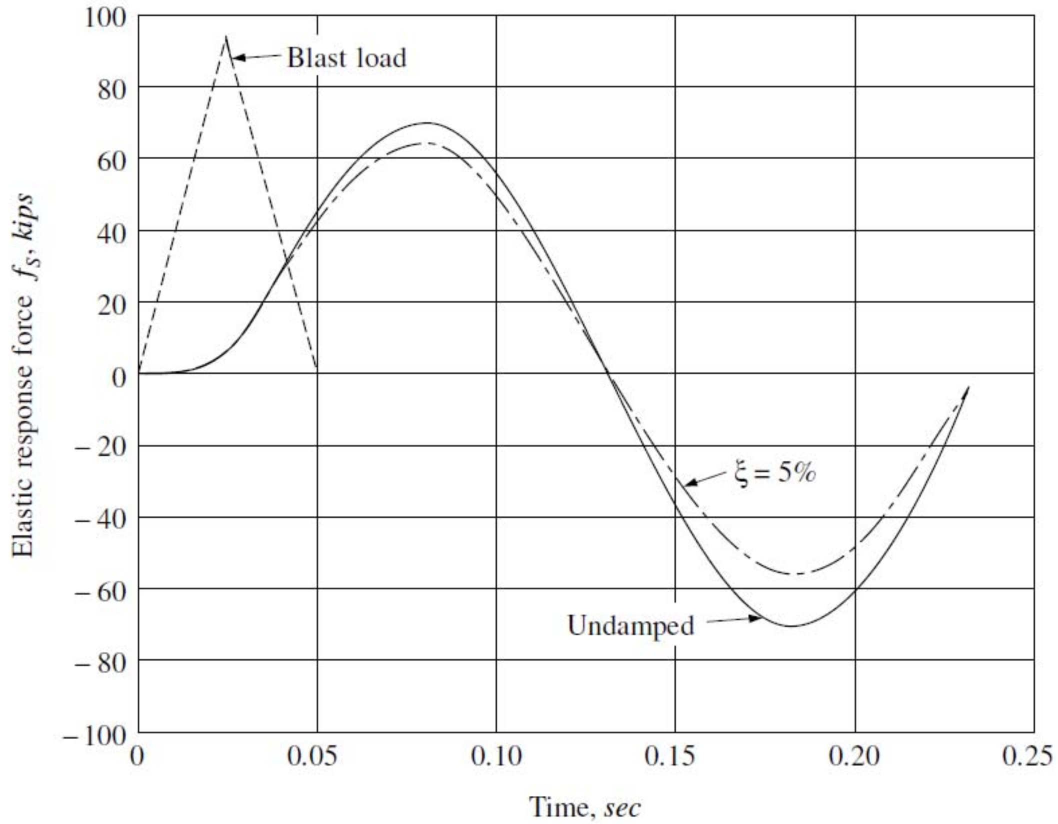
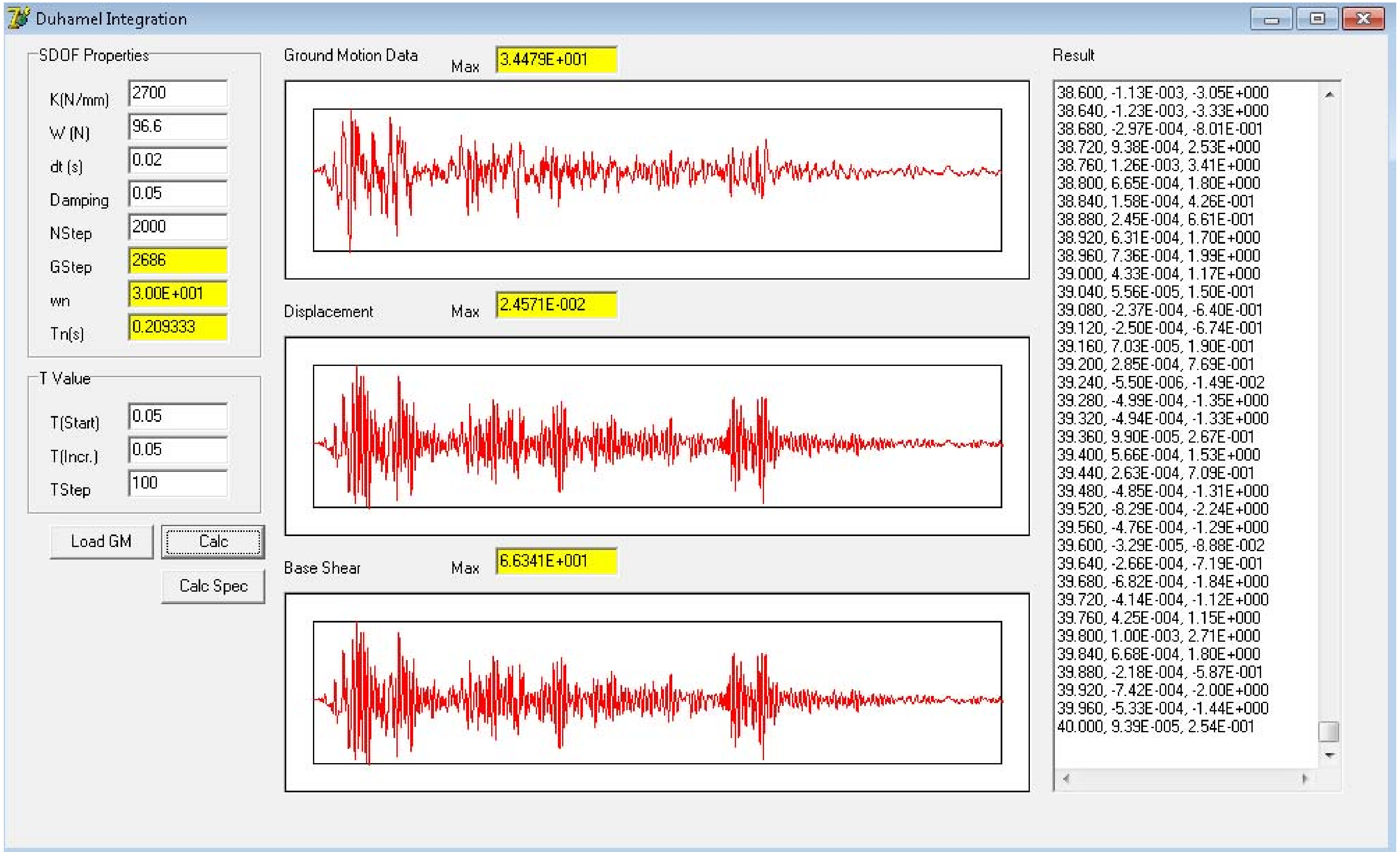


FIGURE E6-2
Response of water tower to blast load.

Program for Duhamel Integration (Solver For Force Vibration Problem of SDOF)



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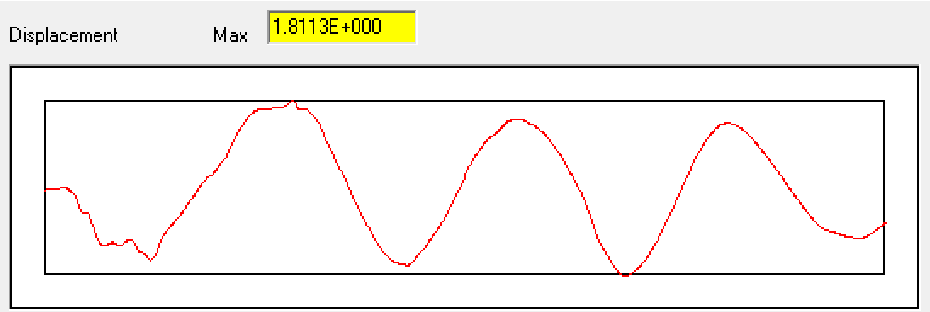
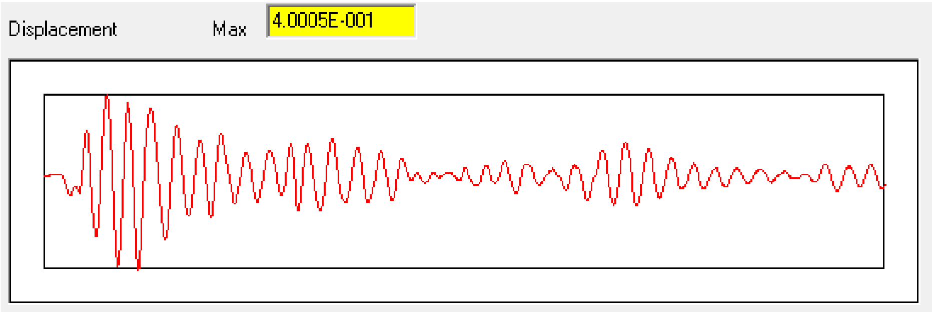
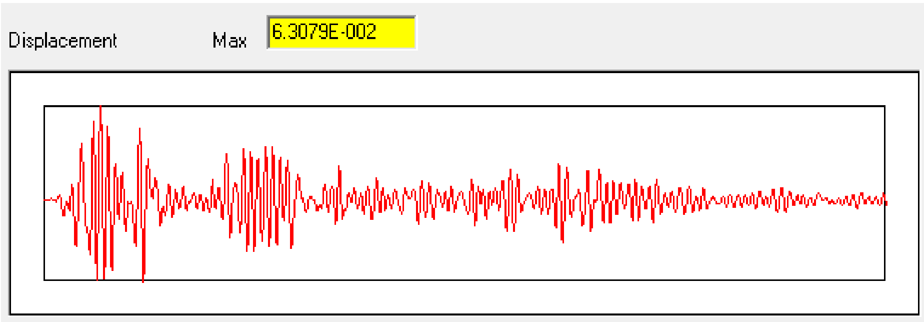
K = 1, T1 = 10.87 s

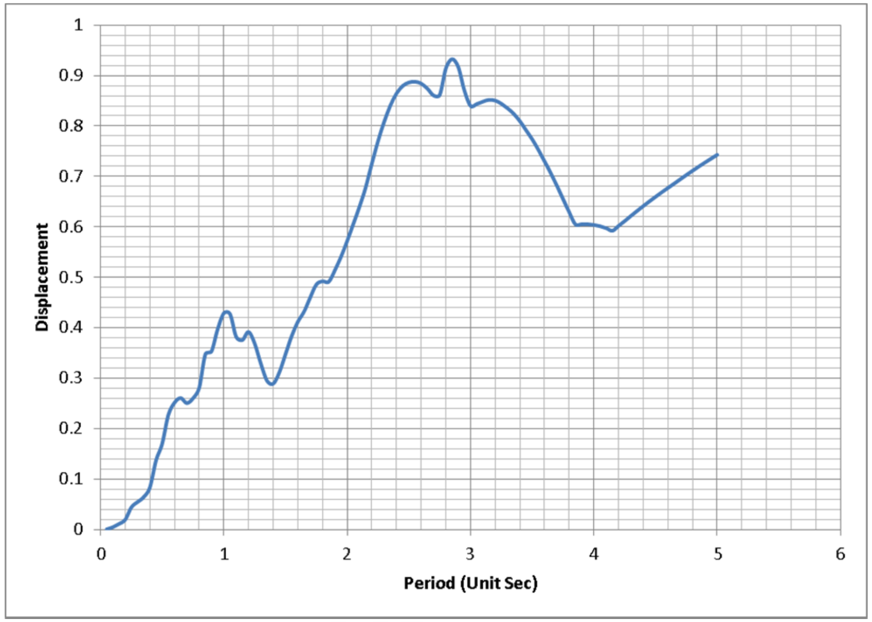


K = 100, T1 = 1.087 s



K = 1000, T1 = 0.344 s





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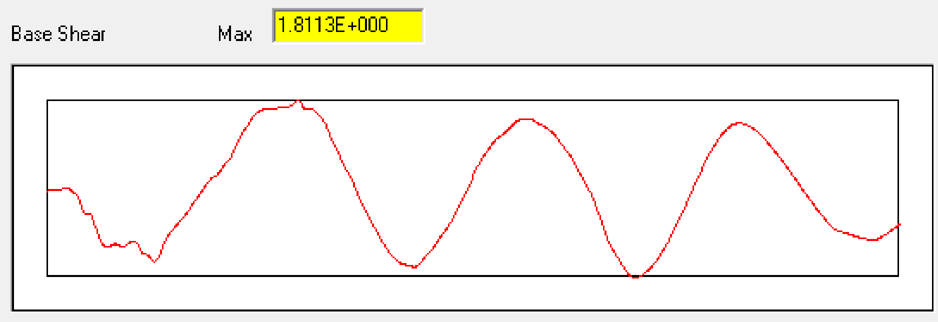

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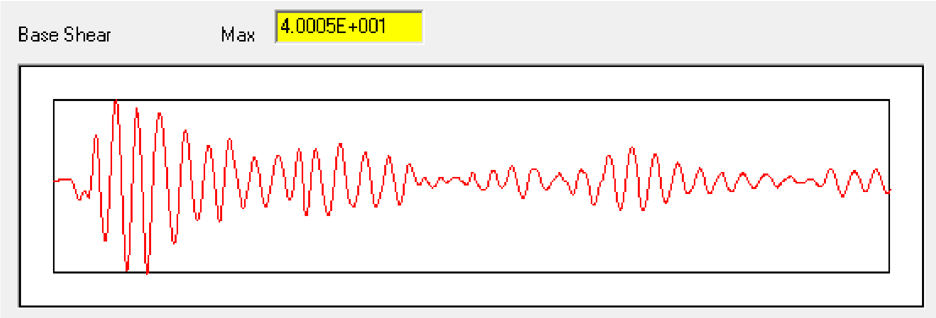

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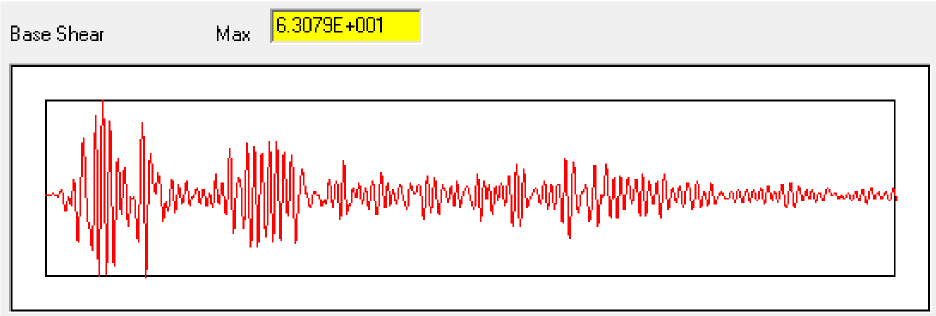

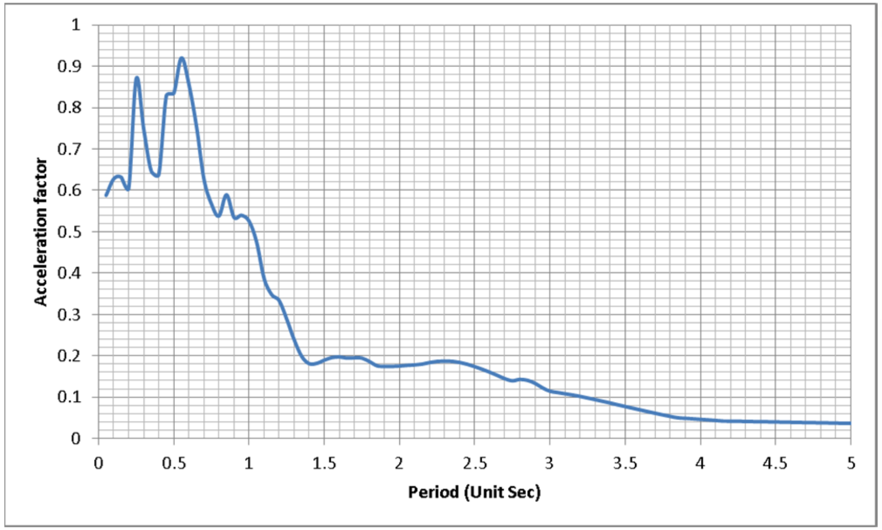

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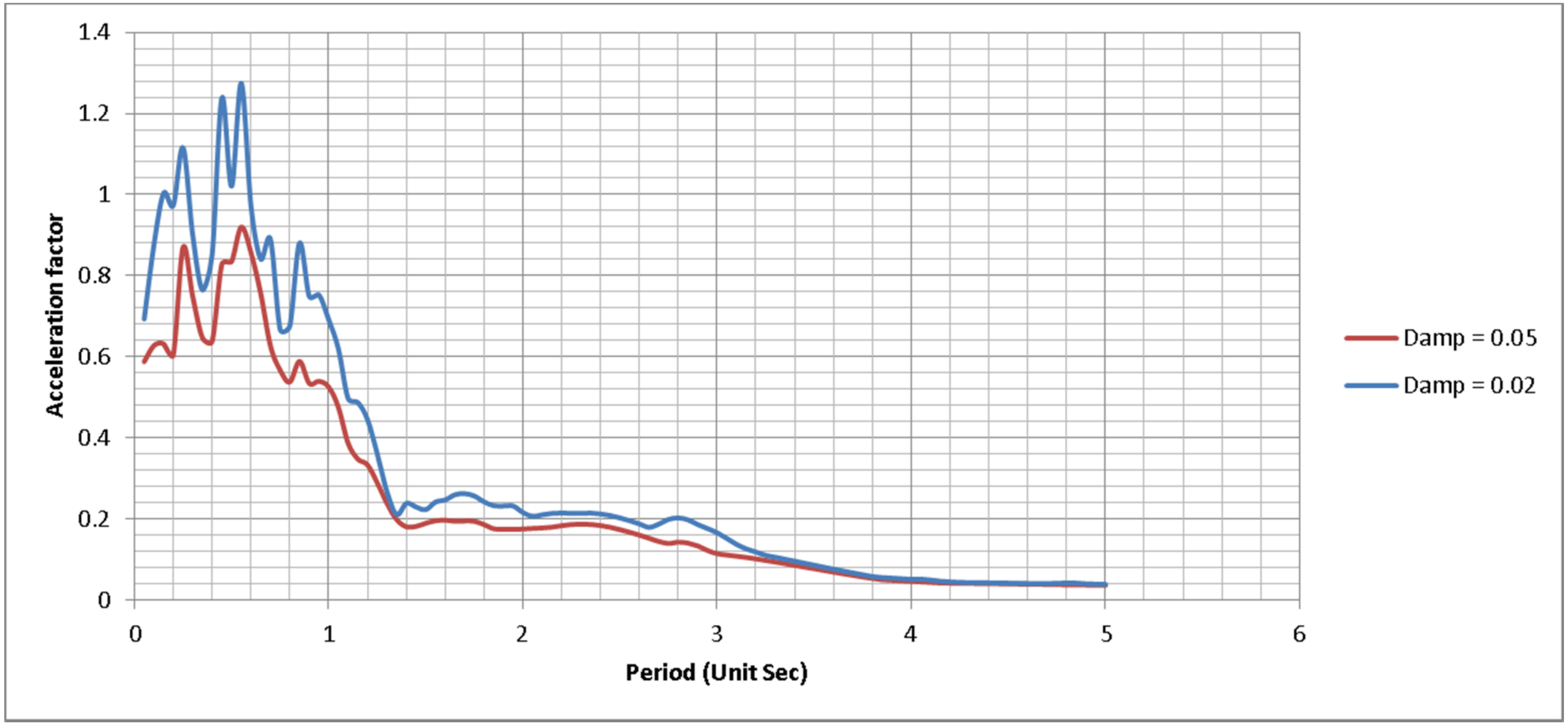
Spec Analysis From Duhamel Integration



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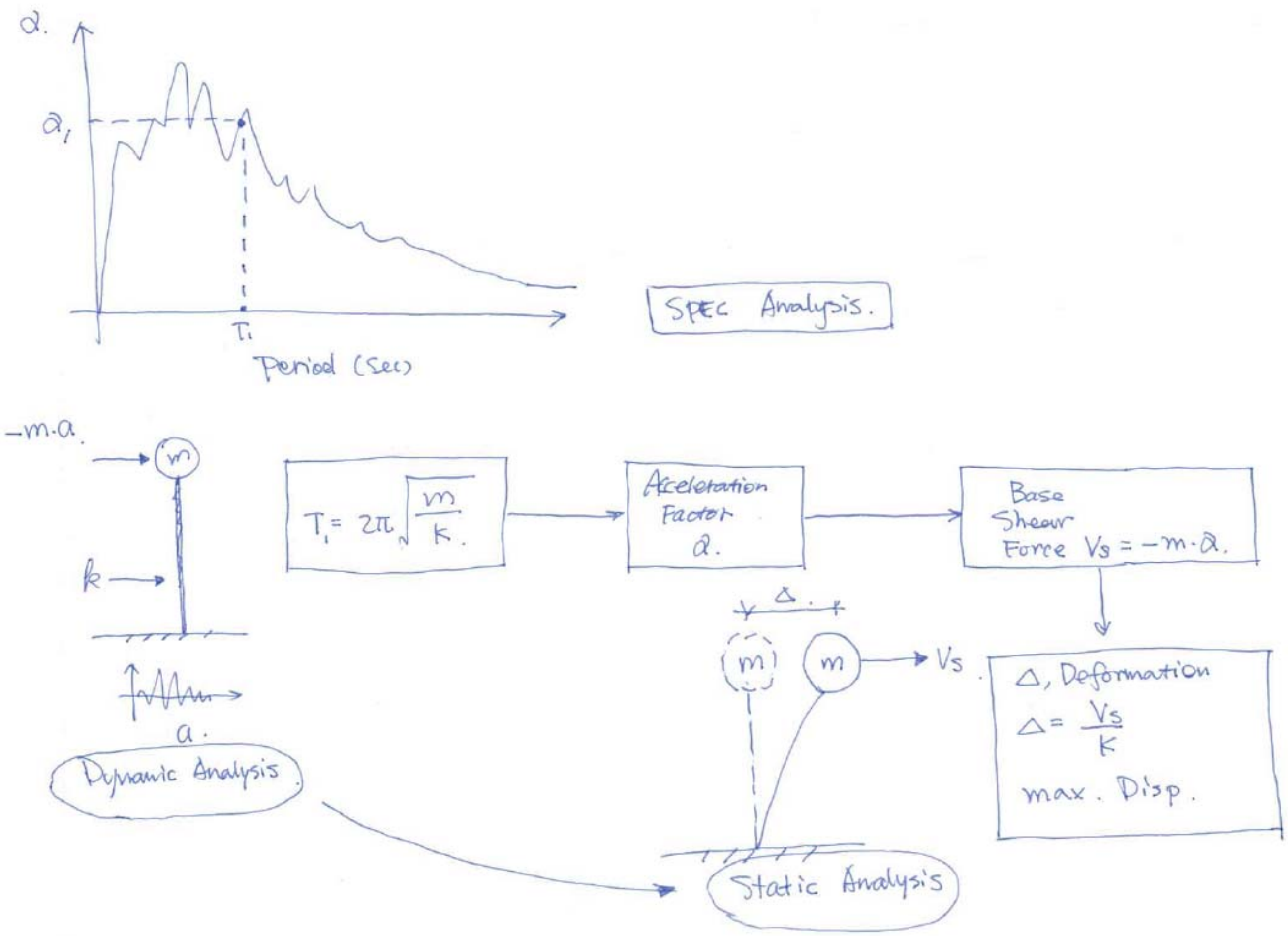
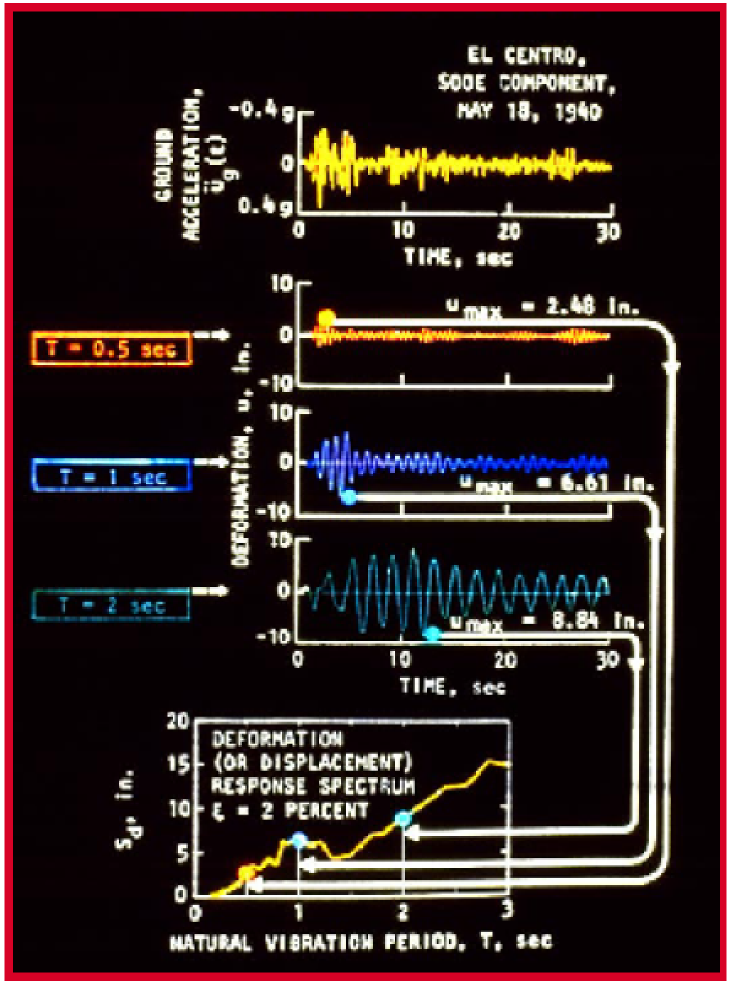
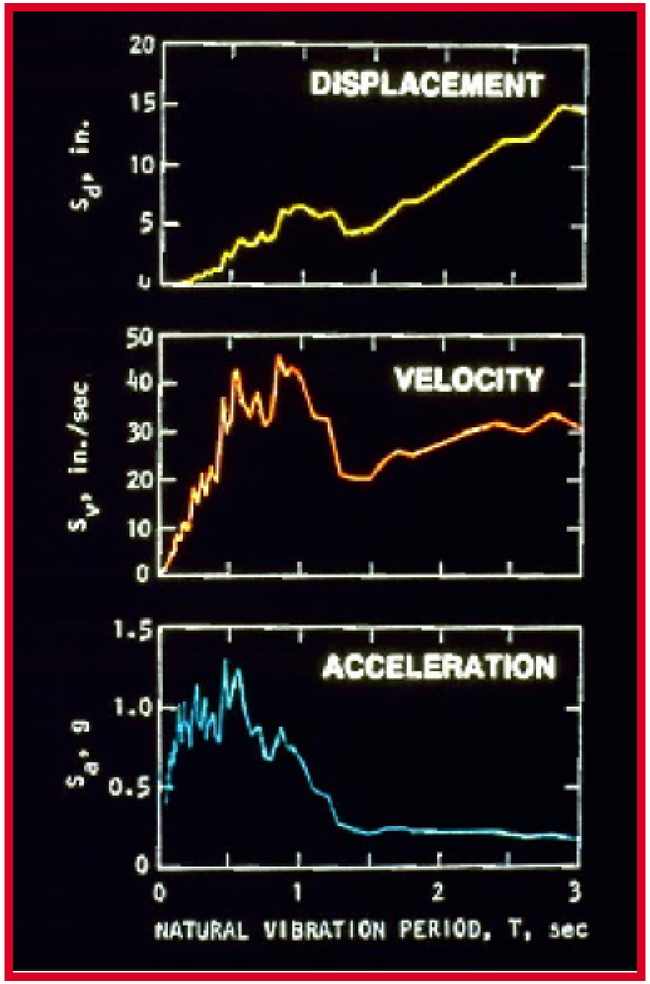
Spec Analysis Simple Theory



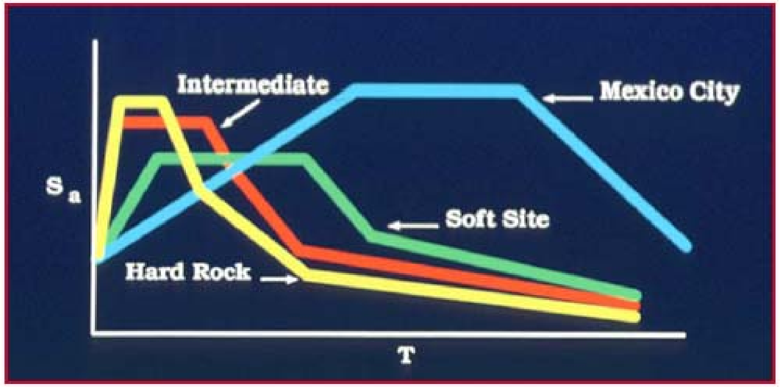
Lec-03 Forced Vibration of SDOF System

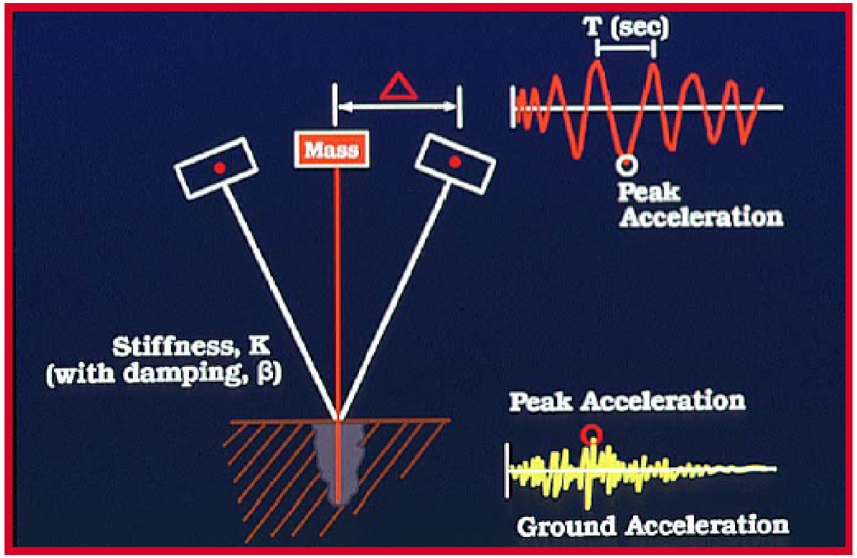
Spec Analysis Simple Theory

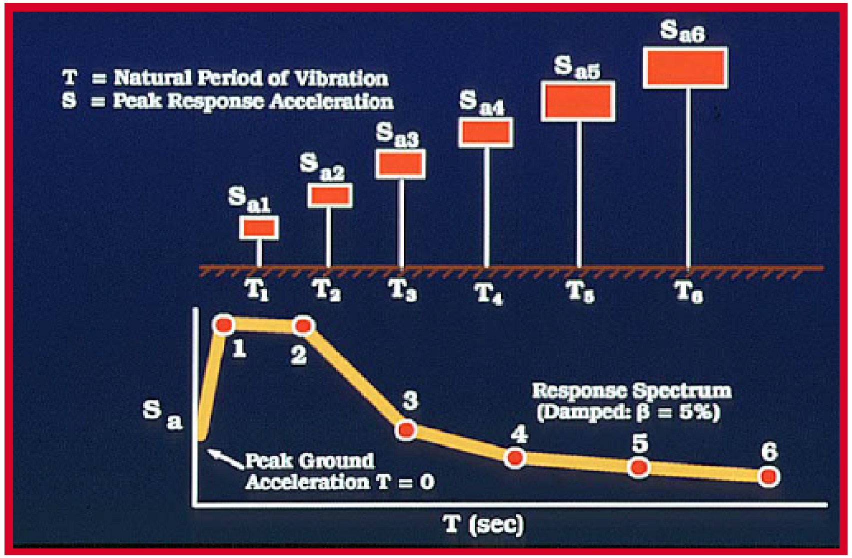




Spec Analysis Simple Theory

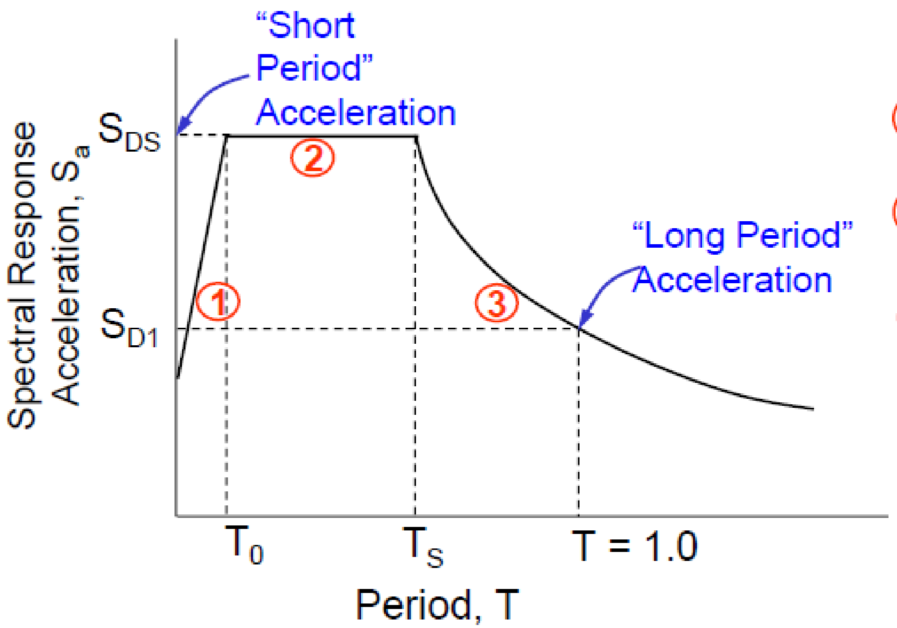






Spec Analysis Simple Theory

ASCE 7-02 Uses a Smoothed Design Acceleration Spectrum



① $S_a = 0.6\dfrac{S_{DS}}{T_0}T + 0.4\,S_{DS}$

② $S_a = S_{DS}$

③ $S_a = \dfrac{S_{D1}}{T}$

$T_0 = \dfrac{0.2\,S_{D1}}{S_{DS}}$

SDOF Simulation Program

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